

Teaching Relativity in Week One

Working with thought experiments
rather than mathematical formalism

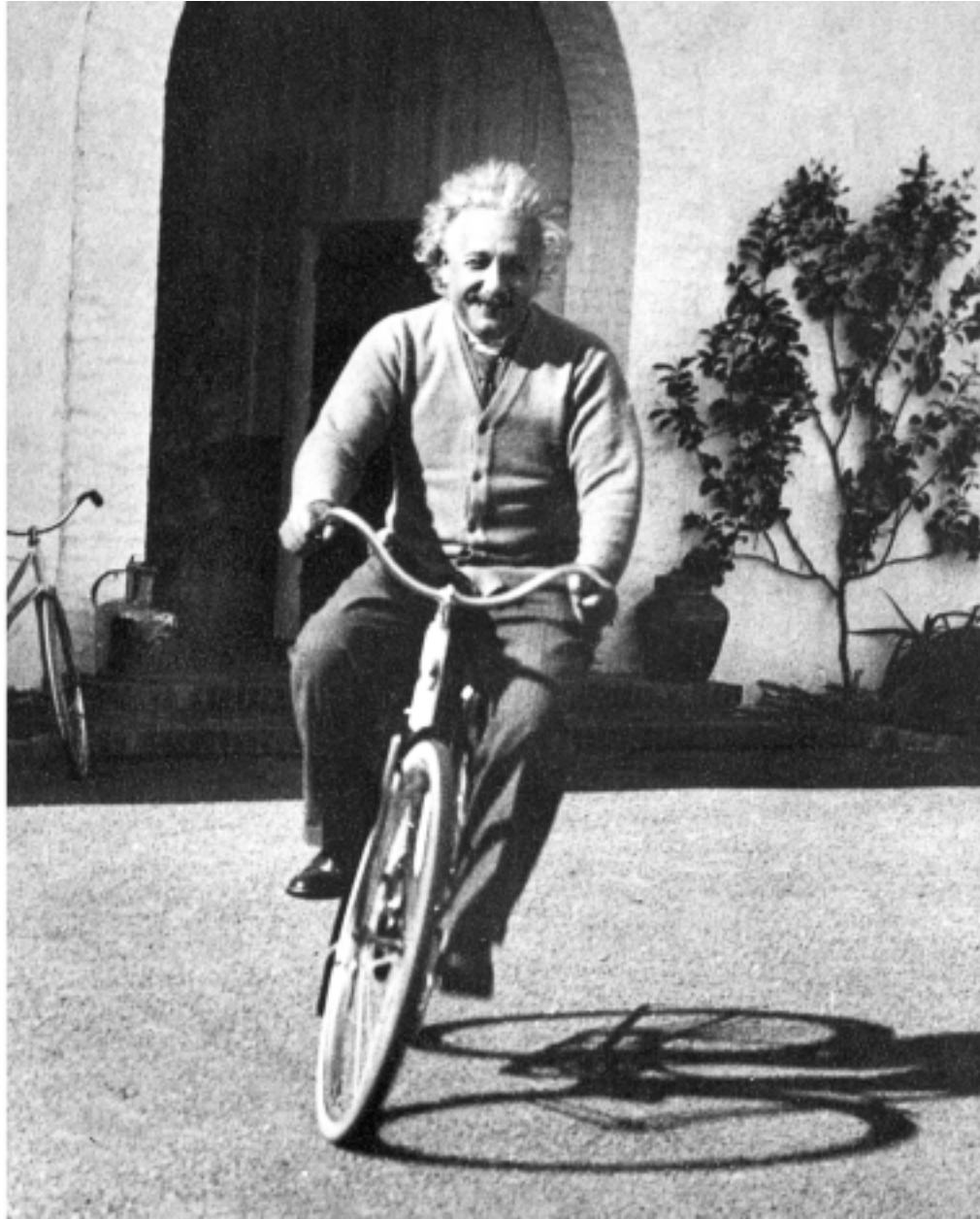


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Based on the text Physics2000 by Prof. E. R. Huggins

Available at **Physics2000.com** – Complete CD \$10
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Aside from the fact that it is an exciting way to begin the course, the advantage of teaching special relativity at the beginning of an introductory physics course is that you can *fit twentieth century concepts into the course as you go along. This eliminates the artificial divide between classical and modern physics.*

The purpose of this pamphlet is to provide a detailed outline of how to teach special relativity in week one using *Chapter 1 of the Physics2000 text* and the *Muon Lifetime Movie* on the *Physics2000 CD*.

Even if you do not use the remainder of the *Physics2000* text, you will see the advantage of starting with special relativity no matter what text you use or what mathematical background your students have. All they need is the Pythagorean theorem and basic algebra.

Schedule for five 45 Minute Periods

Day 1

Principle of relativity

Light waves and Maxwell's theory

Day 2

Einstein's Solution – Speed of light same to all

Light pulse clock runs slow

Day 3

Review

Muon Lifetime movie

GPS

Day 4

Lorentz contraction

A consistent theory

Day 5

Lack of simultaneity

Addenda

Where Special Relativity is used in

Physics2000 Text

Day 1

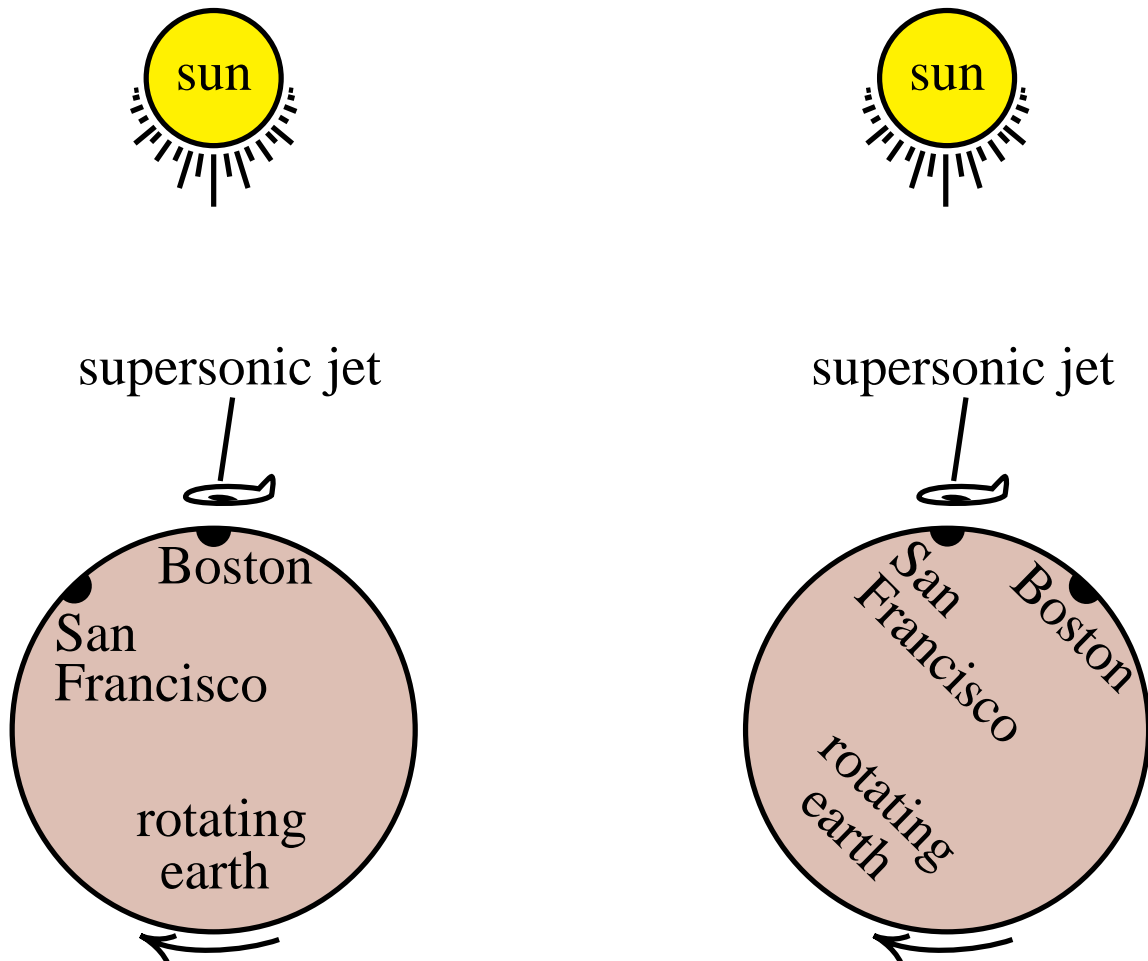
1. The Principle of Relativity

You cannot detect uniform motion

Student experience – Flying in a jet plane. Even though you are flying at 600 miles per hour, the coffee you spill lands in your lap just as it did when the plane was on the ground.

Related thought experiment

Consider a 3-hour trip from Boston to San Francisco on a supersonic jet that leaves at noon. Here is one way to look at it.



Statement of the principle of relativity

Imagine that you are in a capsule and you may have any equipment you wish inside the capsule. The principle of relativity states that *there is no experiment you can perform that will allow you to tell whether or not the capsule is moving with uniform motion* —motion in a straight line at constant speed.

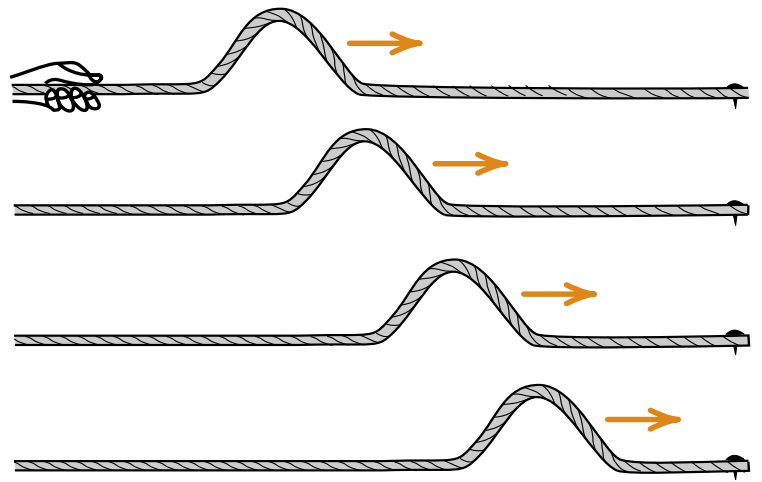
2. Wave Motion

Speed of waves

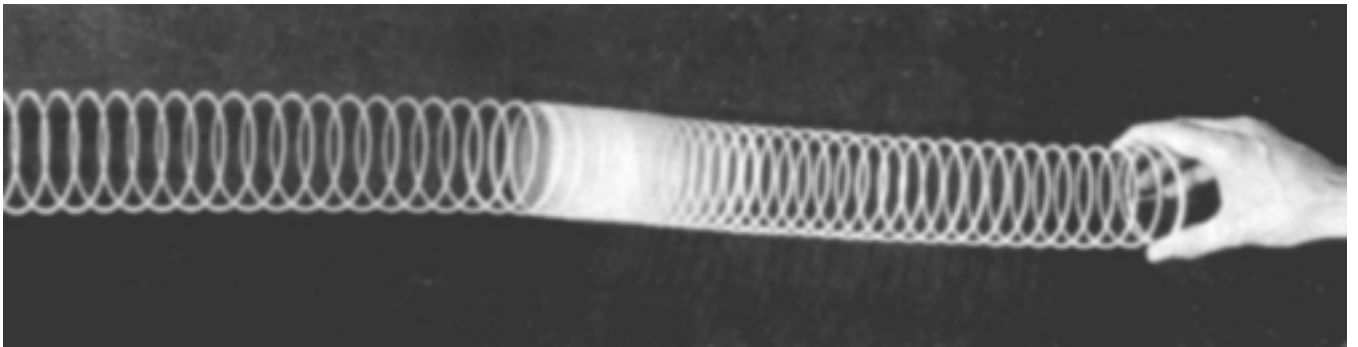
Physicists are good at predicting the speed of waves through a medium.

Rope waves

$$\text{Speed of wave pulse} = \sqrt{\frac{\tau}{\mu}}$$



Slinky and Sound waves

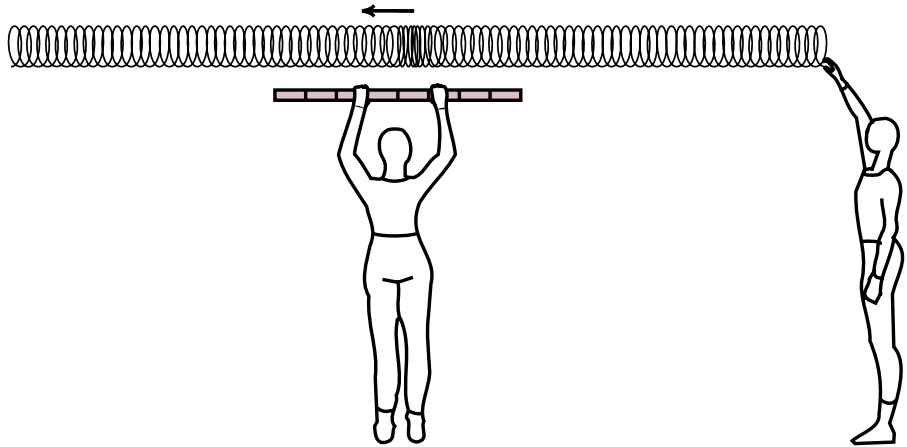


$$\text{Speed of sound} = \sqrt{\frac{B}{\rho}}$$

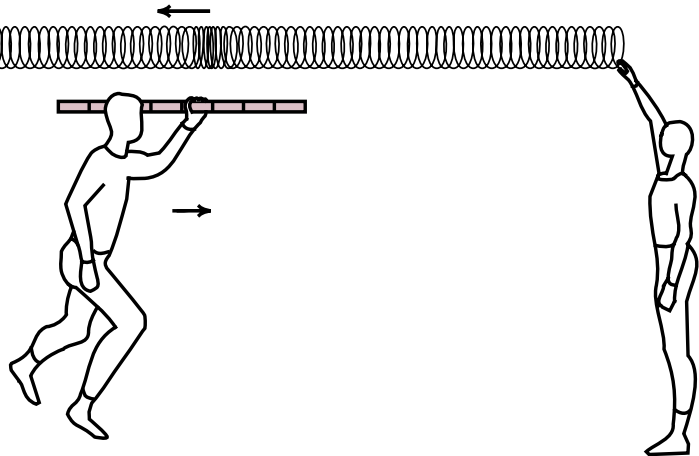
Measuring the speed of a wave

Standing still,
I measure the
predicted speed

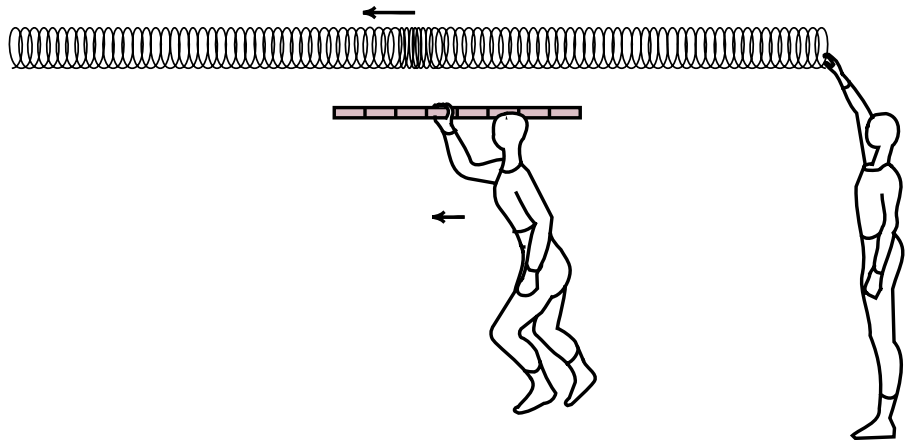
$$v = \sqrt{\frac{B}{\rho}}$$



Moving toward
the wave, I
observe that the
wave passes by
my meter stick
in less time.



When I move
the other way,
the wave takes
longer to pass
by my meter
stick.



Conclusion

Only when I am at rest relative to the Slinky do I observe the wave pass by me at the predicted speed.

Light Waves

In the 1860's James Clerk Maxwell developed the theory of light waves. From the theory, he predicted that light traveled at a speed given by the formula

$$\text{Speed of light } c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

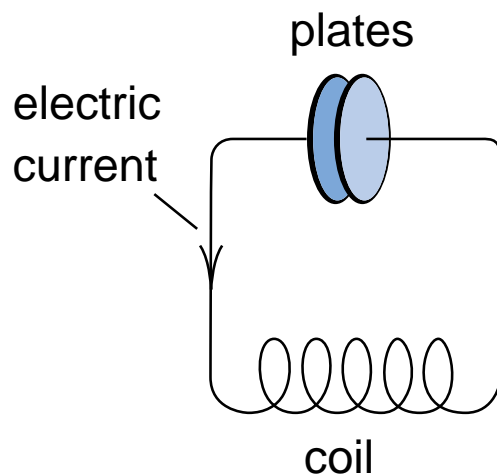
where μ_0 and ϵ_0 are constants in the theory of electricity. The predicted value turns out to be very close to

$c = 1 \text{ foot per nanosecond}$

Every billionth of a second light crawls a distance of one foot. Light takes 1.26 seconds to go from the earth to the moon because the moon is 1.26 billion feet away.

Student Lab

In a second semester lab, students determine the constants μ_0 and ϵ_0 by measuring the rate at which an electric current oscillates between a coil and metal plates. Their answer for c is usually within 10% of 1 foot per nanosecond.



Who gets to measure the predicted speed???

From an experiment that does not look at light, we predict that light should travel at a speed c equal to one foot per nanosecond. Who gets that answer?

With the Slinky wave, only the person at rest relative to the Slinky got the predicted answer.

With Slinky waves, rope waves, water waves and sound waves, only the person at rest relative to the medium of the wave observes waves pass them at the predicted speed.

Light waves travel through a vacuum. Will only those people at rest relative to the vacuum observe light waves pass by them at the predicted speed c ?

Maxwell's theory leads to the conclusion that if you measure a pulse of light passing by you at a speed greater than or less than c , you must be moving relative to the vacuum.

The principle of relativity says that you cannot detect your uniform motion relative to the vacuum.

Are Maxwell's Theory and the Principle of Relativity in conflict?

Tune in tomorrow to find out

Day 2

3. Einstein's Solution

Einstein made Maxwell's theory of light consistent with the principle of relativity by assuming that:

The speed of light is the same for all observers.

Whenever anyone measures the speed of a pulse of light, the result will always be the same value:

$c =$ one foot per nanosecond.

As a result, no measurement of the speed of light can tell you anything about your own motion!

By the way,

1 foot = .3 meters

thus the speed of light is also .3 billion meters per second, or

$c = 3 \times 10^8$ meters per second

4. Consequences of Einstein's Assumptions

There are really two assumptions,

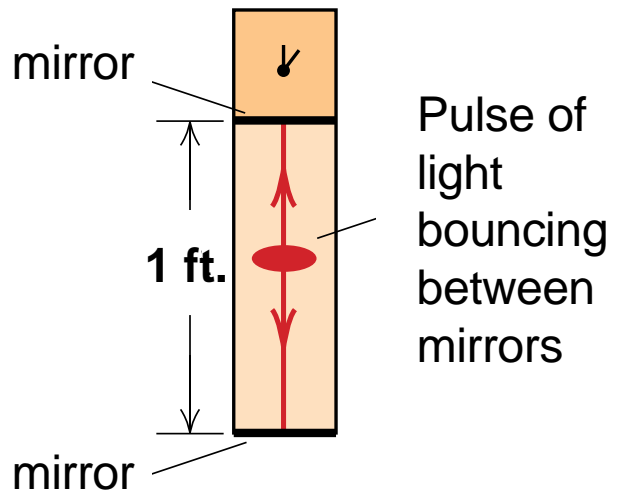
- 1) The principle of relativity is correct
- 2) The speed of light is the same to all observers

How do we figure out what these assumptions imply?

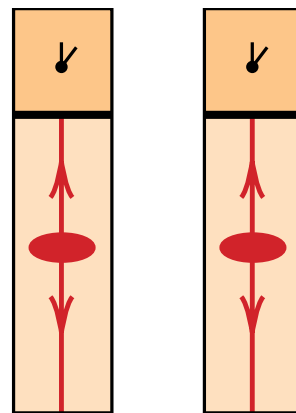
We use thought experiments!

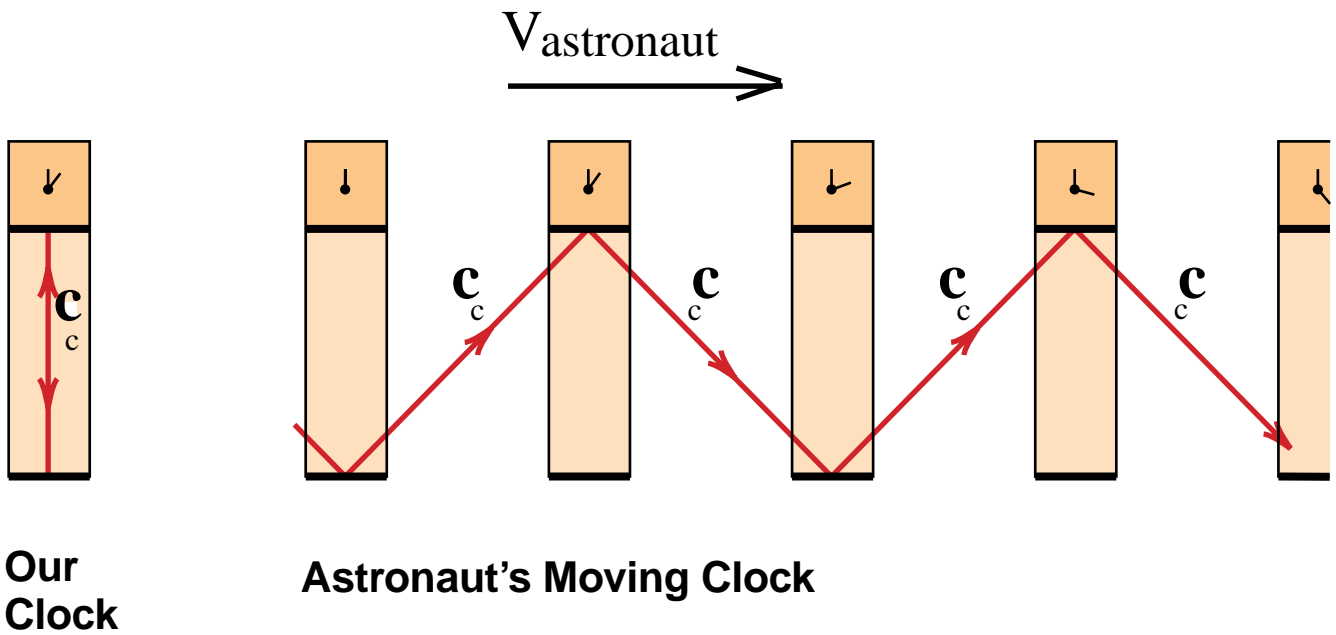
4a. The light pulse clock

Make a clock whose timing is based on a pulse of *light bouncing between two mirrors*. The clock face just counts bounces, which will be nanoseconds if the mirrors are one foot apart.

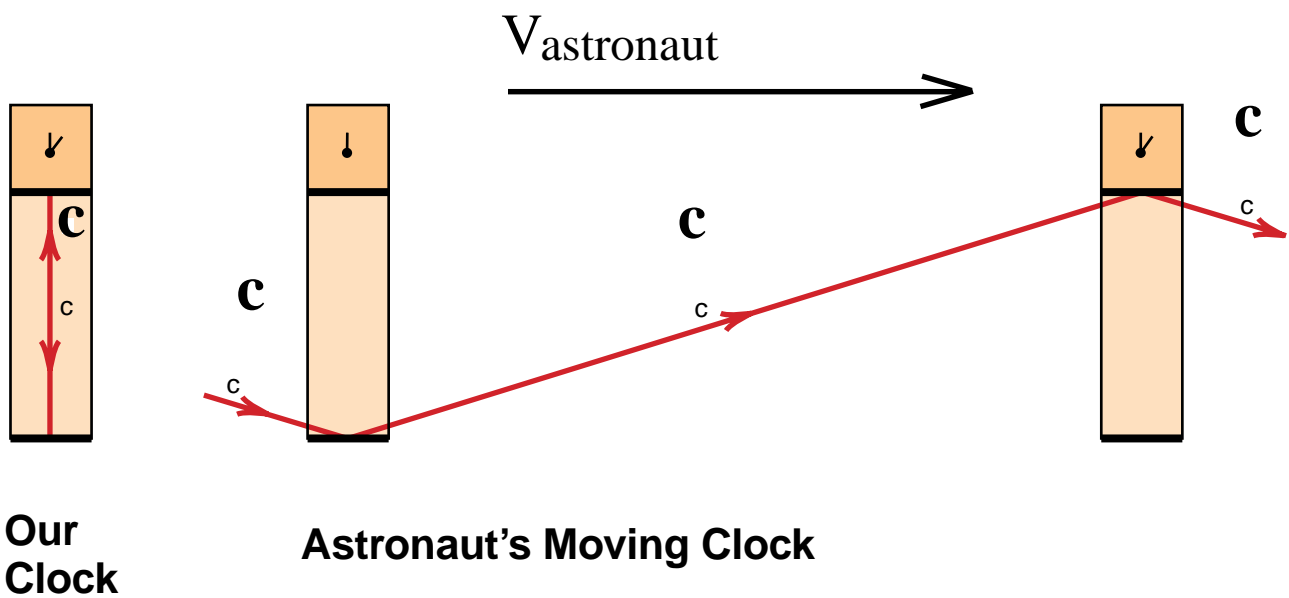


For this thought experiment, we make two identical clocks. We keep one, and give the other to an astronaut who will go rushing by us at a speed near the speed of light.



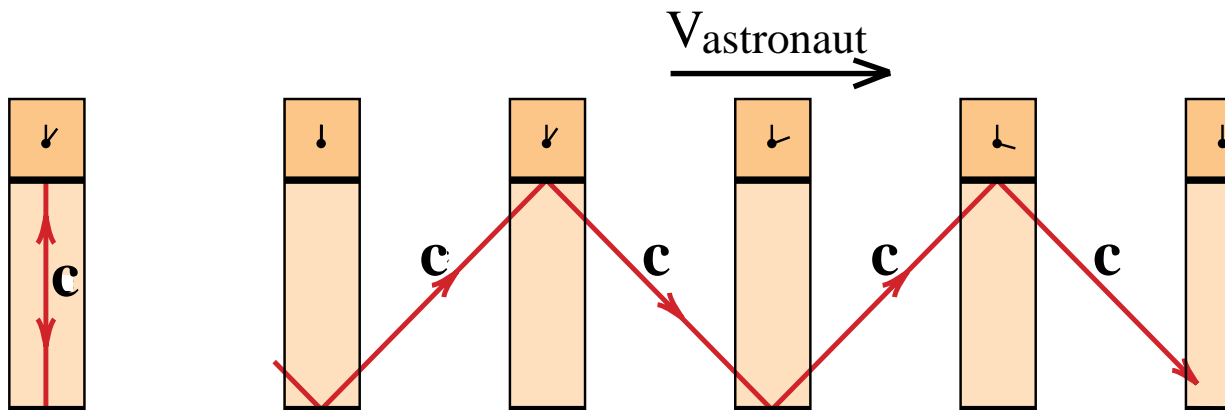


In order to stay in the astronaut's clock, the light pulse must follow the longer, saw-tooth path.

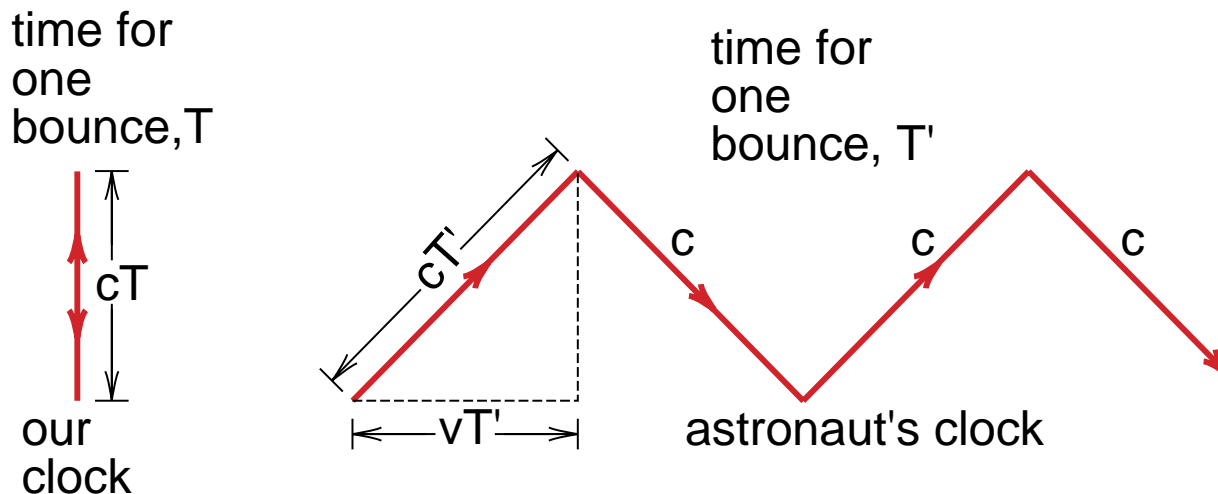


When the astronaut goes faster, his light pulse has to go farther in order to register a bounce. Since the speed of light does not change, it takes longer for one bounce to register, and the astronaut's moving clock runs slower.

4b. The Math!!!



Let T be the time for one bounce in our clock. The astronaut's clock takes a longer time T' .



The dotted triangle has sides vT' , cT , and a hypotenuse cT' . The **Pythagorean theorem** tells us that

$$(cT')^2 = (vT')^2 + (cT)^2 \quad ; \quad T'^2 = \frac{T^2}{1 - v^2/c^2}$$

$$T' = \frac{T}{\sqrt{1 - v^2/c^2}}$$

Day 3

4c. Time Dilation

The light pulse clock has told us that if Einstein's two assumptions are correct, then the astronaut's seconds are longer than ours by a factor

$$\frac{1}{\sqrt{1 - v^2/c^2}}$$

or that his clock runs slow by a factor $\sqrt{1 - v^2/c^2}$. As the astronaut's speed approaches the speed of light, $1 - v^2/c^2$ goes to zero, and the astronaut's clock stops!

Does this really happen? Yes!

The Time Dilation movie studies the lifetime of muons traveling at a speed $v = .994c$, and shows that the moving muons live nine times longer than muons at rest.



Exercise:

Show that

$$1/\sqrt{1 - v^2/c^2} = 9.1$$

for

$$v = .994c$$

(use a calculator)

Scene from the 36 minute Movie *Time Dilation: An Experiment with Mu-Mesons*. You will find this movie in the Movies folder of the **Physics2000 CD**.

***Click on the movie and run it now.
(See instructions on the back of
this pamphlet.)***

4d. Is Time Dilation of Any Practical Importance? YES!

A satellite in orbit, traveling nearly 17,000 miles per hour (27000 km/hr), loses only 7 millionths of a second a day due to time dilation. Could anyone care about this 7 microsecond loss per day?

The answer is that **you do!**

GPS

GPS receivers determine your position by measuring the length of time it takes radio signals to travel from at least 4 GPS satellites* to your receiver. This determines your position relative to the satellites, because the radio signals (a form of light) travel at the speed c , which is **1000 feet per microsecond**. The timing is based upon atomic clocks in the satellites.

If one of the atomic clocks were off by just one microsecond, that would lead to a 1000 foot error in determining your location. If the designers of the GPS system did not take Einstein's **Time Dilation** into account, the system would **accumulate an error of 7,000 feet, or 1.3 miles per day**. The GPS system would become useless within a few minutes.

*Four satellites are needed to establish your GPS's space-time coordinates x, y, z, t .

Day 4

5. Lorentz Contraction

Muon's clocks run slow

In the muon lifetime movie, the muons made it down from the top of Mt. Washington to sea level because their clocks ran slow.

(Muons at rest live only about 2 microseconds. The 6,000 foot trip down from Mt. Washington takes 6 microseconds when traveling at nearly the speed of light c . Thus the moving muons could not have survived the trip unless they lived longer, unless their clocks ran slow.)

No, the mountain is short.

From the muon's point of view, they are sitting at rest and the mountain is moving by. Muons at rest live 2 microseconds. Since the mountain went by before the muons decayed, the mountain went by in less than 2 microseconds, traveling at a speed of nearly $c = 1000$ feet per microsecond. In 2 microseconds, only 2,000 feet of mountain could go by. Since the whole mountain went by, the mountain is less than 2,000 feet tall.

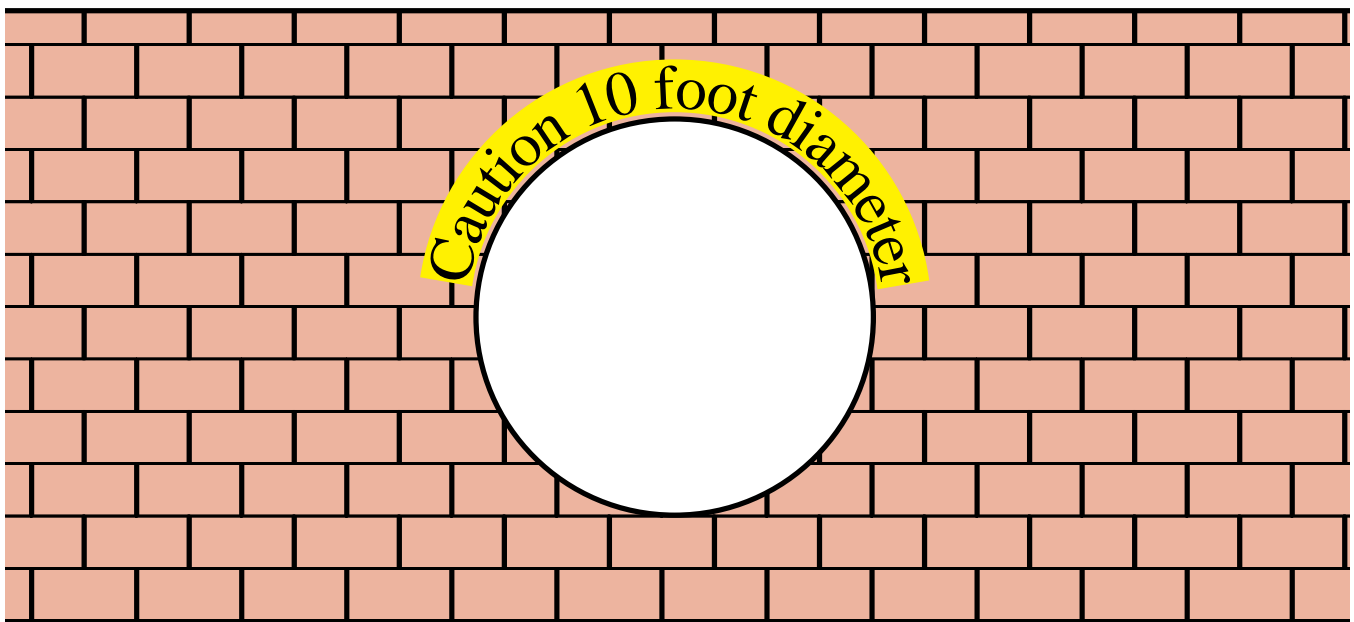
Lorentz contraction Formula

For the muons, the height of Mt. Washington must contract by the same factor $\sqrt{1 - v^2/c^2}$ that we say the rate of the muon's clock slowed. In general, the length of all moving objects contract by this same factor. Thus the formula is for the length of an object, in its direction of motion, is:

$$L' = L \times \sqrt{1 - v^2/c^2}$$

Do widths contract?

As a thought experiment, we send an astronaut in a 10 foot diameter capsule through a 10 foot diameter hole in a wall. A dramatic contradiction results, demonstrating that widths do not contract.



6. A Consistent Theory

In the muon lifetime movie, there is an important experimental fact. Most of the muons survived the trip down from the top of Mt. Washington. We all have to agree to that. But we do not have to agree on how the muons survived.

We say the muons survived because their clocks ran slow.

The muons say they survived because the mountain was short.

In special relativity, you have to agree on the experimental results, but you can have different points of view of how things happened.

We need to study one more effect to obtain a consistent theory, where everyone agrees on experimental results. The third new effect is the **Lack of Simultaneity**.

Your Schedule

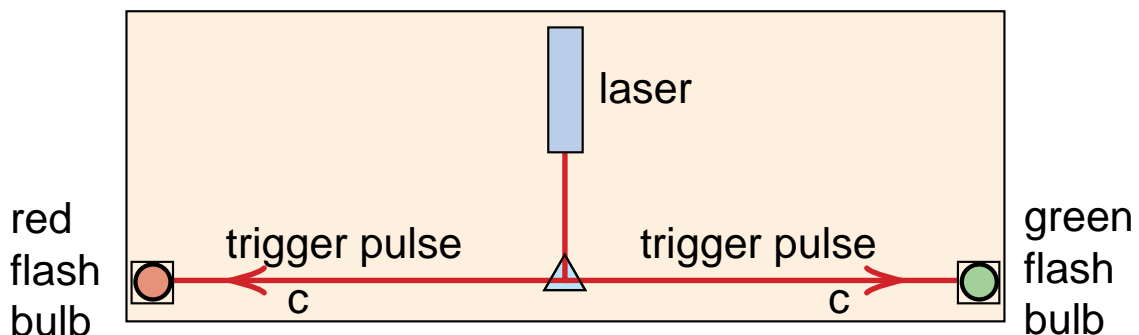
If you have gotten here at the end of week one, you are doing fine. The next topic, the lack of simultaneity is a lot of fun, but not essential for the modern physics topics that should be brought into an introductory physics course.

If you did special relativity in week one, there should be time as the course goes on to return to this topic.

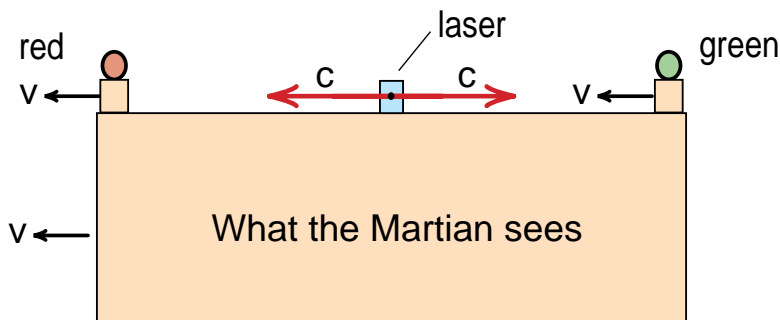
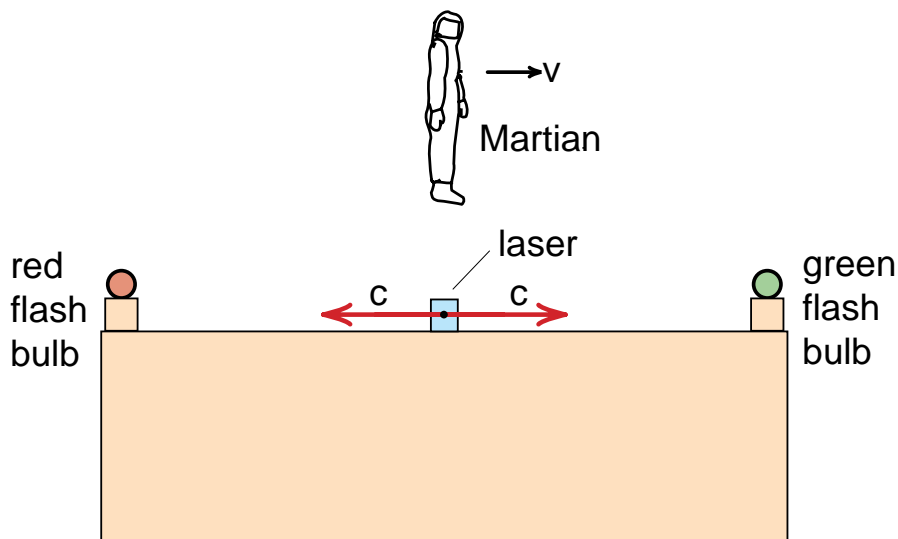
Day 5

7. Lack of Simultaneity

Two events that for us were simultaneous, will generally not be simultaneous events to someone moving by. Here is a **thought experiment** that we actually carry out in class.

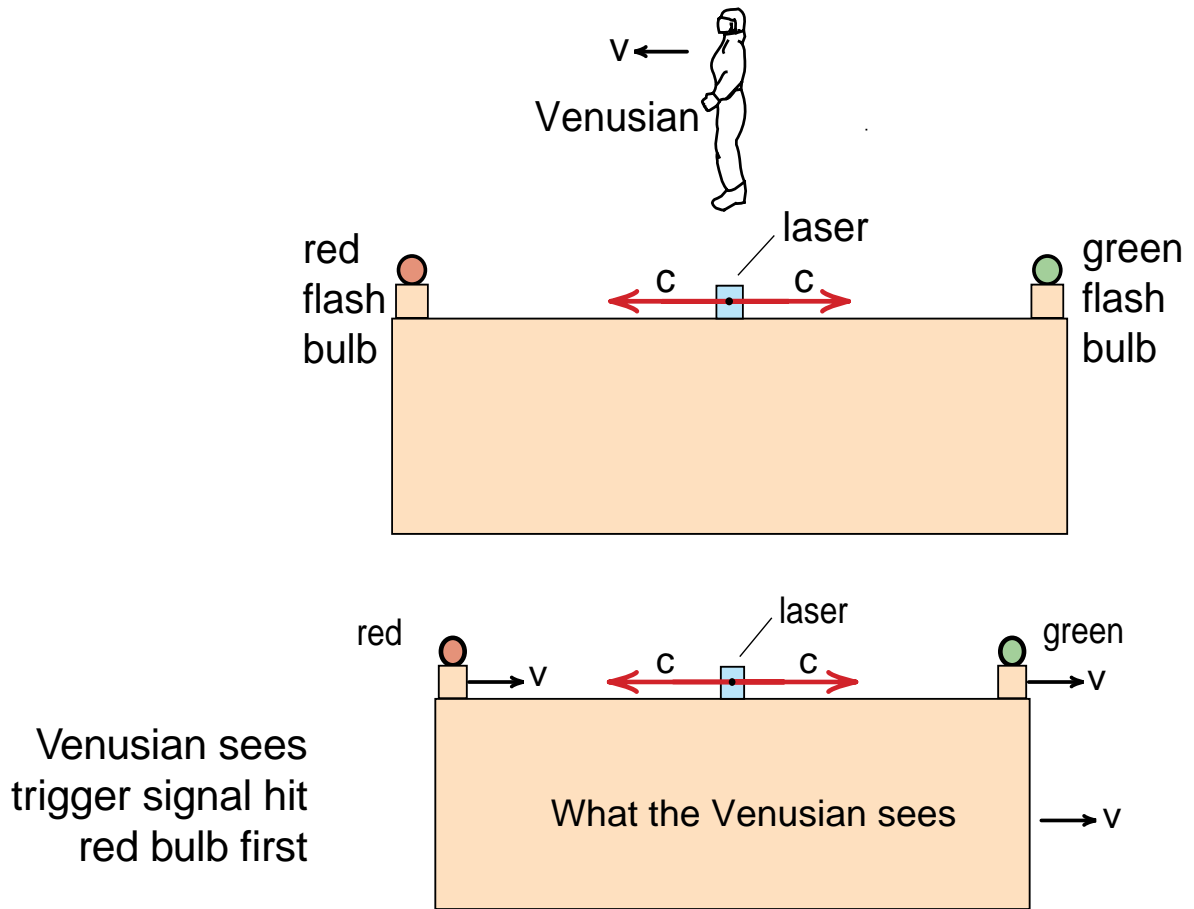


Martian's view of our thought experiment



Martian sees trigger signal hit green bulb first

Venusian's view of our thought experiment



The order of two events can depend on the observer's point of view.

We show that to preserve cause and effect (for example, if you cannot get the answer to a question that has not been thought of yet), then information cannot travel faster than the speed of light.

Addenda

8. Using Special Relativity

If you start in week one with special relativity,

- a) The students have the rest of the course to become familiar with the strange but exciting concepts of special relativity.**

- b) You can introduce 20th century concepts as you go along.**

“Modern Physics” can be integrated into the course, not put off to where few students find it.



9. Where Special Relativity is used in Physics2000

Mass We use an experimental recoil definition of mass (Page 6-4). A simple thought experiment immediately demonstrates that mass has to increase with velocity as the speed of the object approaches c . This leads to a discussion of Einstein's relativistic mass formula.

Energy We can begin our discussion of energy with $E = mc^2$. Kinetic energy arises from the increase of mass with velocity (Page 10-5).

Magnetism We use Coulomb's Law and the Lorentz contraction to derive the magnetic force law and the formula for the magnetic field of a straight wire (Page 28-4).

Photons We have an easy chapter on Photon Mechanics (Chapter 34) following the discussion of light waves.

Uncertainty Principle From the fact that light is both a particle and a wave, using Fourier analysis we show that the probability interpretation of the wave leads to the Uncertainty Principle (Page 40-19)



The Physics2000 text is available at ***Physics2000.com***
Complete CD \$10

Physics2000

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Calculus Ch 1 INTRODUCTION TO CALCULUS

Chapter 1

Principle of Relativity

The subject of this book is the behavior of matter—the particles that make up matter, the interactions between particles, and the structures that these interactions create. There is a wondrous variety of activity, as patterns and structures form and dissipate, and all of this activity takes place in an arena we call space and time. The subject of this chapter is that arena—space and time itself.

*Initially, one might think that a chapter on space and time would either be extraordinarily dull, or too esoteric to be of any use. From the **it's too dull** point of view, distance is measured by meter sticks, and there are relationships like the Pythagorean theorem and various geometric and trigonometric rules already familiar to the reader. Time appears to be less challenging—it is measured by clocks and seems to march inexorably forward.*

*On the **too esoteric** side are the theories like Einstein's General Theory of Relativity which treats gravity as a distortion of space and time, the Feynman-Wheeler picture of antimatter as being matter traveling backward in time, and recent "super symmetry" theories which assume a ten dimensional space. All of these theories are interesting, and we will briefly discuss them. We will do that later in the text after we have built up enough of a background to understand why these theories were put forth.*

What can we say in an introductory chapter about space and time that is interesting, or useful, or necessary for a physics text? Why not follow the traditional

approach and begin with the development of Newton's theory of mechanics. You do not need a very sophisticated picture of space and time to understand Newtonian mechanics, and this theory explains an enormous range of phenomena, more than you can learn in one or several years. There are three main reasons why we will not start off with the Newtonian picture. The first is that the simple Newtonian view of space and time is approximate, and the approximation fails badly in many examples we will discuss in this text. By starting with a more accurate picture of space and time, we can view these examples as successful predictions rather than failures of the Newtonian theory.

The second reason is that the more accurate picture of space and time is based on the simplest, yet perhaps most general law in all of physics—the principle of relativity. The principle of relativity not only underlies all basic theories of physics, it was essential in the discovery of many of these theories. Of all possible ways matter could behave, only a very, very few are consistent with the principle of relativity, and by concentrating on these few we have been able to make enormous strides in understanding how matter interacts. By beginning the text with the principle of relativity, the reader starts off with one of the best examples of a fundamental physical law.

Our third reason for starting with the principle of relativity and the nature of space and time, is that it is fun. The math required is simple—only the Pythagorean theorem. Yet results like clocks running slow, lengths contracting, the existence of an ultimate speed, and

questions of causality, are stimulating topics. Many of these results are counter intuitive. Your effort will not be in struggling with mathematical formulas, but in visualizing yourself in new and strange situations. This visualization starts off slowly, but you will get used to it and become quite good at it. By the end of the course the principle of relativity, and the consequences known as Einstein's special theory of relativity will be second nature to you.

THE PRINCIPLE OF RELATIVITY

In this age of jet travel, the principle of relativity is not a strange concept. It says that *you cannot feel motion in a straight line at constant speed*. Recall a smooth flight where the jet you were in was traveling at perhaps 500 miles per hour. A moving picture is being shown and all the window shades are closed. As you watch the movie are you aware of the motion of the jet? Do you feel the jet hurtling through the air at 500 miles per hour? Does everything inside the jet crash to the rear of the plane because of this immense speed?

No—the only exciting thing going on is the movie. The smooth motion of the jet causes no excitement whatsoever. If you spill a diet Coke, it lands in your lap just as it would if the plane were sitting on the ground. The problem with walking around the plane is the food and drink cart blocking the aisles, not the motion of the plane. Because the window shades are closed, you cannot even be sure that the plane is moving. If you open your window shade and look out, and if it is daytime and clear, you can look down and see the land move by. Flying over the Midwestern United States you will see all those square 40 acre plots of land move by, and this tells you that you are moving. If someone suggested to you that maybe the farms were moving and you were at rest, you would know that was ridiculous, the plane has the jet engines, not the farms.

Despite the dull experience in a jumbo jet, we often are able to sense motion. There is no problem in feeling motion when we start, stop, or go around a sharp curve. But starting, stopping, and going around a curve are not examples of motion at constant speed in a straight line, the kind of motion we are talking about. ***Changes in speed or in direction of motion are called accelerations***, and we can feel accelerations. (Note: In physics a decrease in speed is referred to as a negative acceleration.)

Even without accelerations, even when we are moving at constant speed in a straight line, we can have a strong sense of motion. Driving down a freeway at 60 miles per hour in a low-slung, open sports car can be a notable, if not scary, experience.

This sense of motion can be misleading. The first wide screen moving pictures took the camera along on a roller coaster ride. Most people in the audience found watching this ride to be almost as nerve wracking as actually riding a roller coaster. Some even became sick. Yet the audience was just sitting at rest in the movie theater.

Exercise 1

Throughout this text we will insert various exercises where we want you to stop and think about or work with the material. At this point we want you to stop reading and think about various times you have experienced motion. Then eliminate all those that involved accelerations, where you speeded up, slowed down, or went around a curve. What do you have left, and how real were the sensations?

One of my favorite examples occurred while I was at a bus station in Boston. A number of busses were lined up side by side waiting for their scheduled departure times. I recall that after a fairly long wait, I observed that we were moving past the bus next to us. I was glad that we were finally leaving. A few seconds later I looked out the window again; the bus next to us had left and we were still sitting in the station. I had mistaken that bus's motion for our own!

A Thought Experiment

Not only can you feel accelerated motion, you can easily see relative motion. I had no problem seeing the bus next to us move relative to us. My only difficulty was in telling whether they were moving or we were moving.

An example of where it is more obvious who is moving is the example of the jet flying over the Midwestern plains. In the daytime the passengers can see the farms go by; it is easy to detect the relative motion of the plane and the farms. And it is quite obvious that it is the plane moving and the farms are at rest. Or is it?

To deal with this question we will go through what is called a **thought experiment** where we solve a problem by imagining a sometimes contrived situation, and then figure out what the consequences would be if we were actually in that situation. Galileo is well known for his use of thought experiments to explain the concepts of the new mechanics he was discovering.

For our thought experiment, imagine that we are going to take the Concorde supersonic jet from Boston, Massachusetts to San Francisco, California. The jet has been given special permission to fly across the country at supersonic speeds so that the trip, which is scheduled to leave at noon, takes only three hours.

When we arrive in San Francisco we reset our watches to Pacific Standard Time to make up for the 3 hour difference between Boston and San Francisco. We reset our watches to noon. When we left, it was noon and the sun was overhead. When we arrive it is still noon and the sun is still overhead. One might say that the jet flew fast enough to follow the sun, the 3 hour trip just balancing the 3 hours time difference.

But there is another view of the trip shown in Figure (1). When we took off at noon, the earth, the airplane and sun were lined up as shown in Figure (1a). Three hours later the earth, airplane and sun are still lined up as shown in Figure (1b). The only difference between (1a) and (1b) is that the earth has been rotating for three hours so that San Francisco, rather than Boston is now under the plane. The view in Figure (1) is what an astronaut approaching the earth in a spacecraft might see.

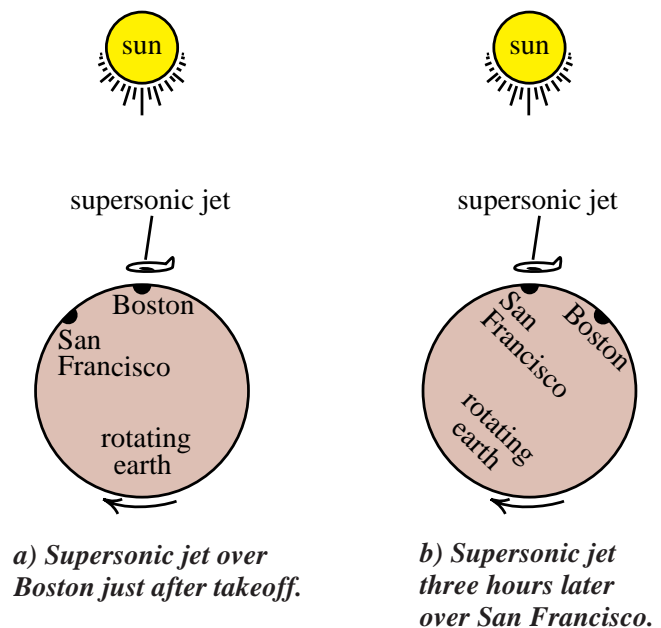


Figure 1

One view of a three hour trip from Boston to San Francisco. It is possible, even logical, to think of the jet as hovering at rest while the earth turns underneath.

For someone inside the jet, looking down at the Midwestern farms going by, who is really moving? Are the farms *really* at rest and the plane moving? Or is the plane at rest and the farms going by? Figure (1) suggests that the latter point of view may be more accurate, at least from the perspective of one who sees the bigger picture including the earth, airplane, and sun.

But, you might ask, what about the jet engines and all the fuel that is being expended to move the jet at 1000 miles/hour? Doesn't that prove that it is the jet that is moving? Not necessarily. When the earth rotates, it drags the atmosphere around with it creating a 1000 mi/hr wind that the plane has to fly through in order to stand still. Without the jet engines and fuel, the plane would be dragged back with the land and never reach San Francisco.

This thought experiment has one purpose. To loosen what may have been a firmly held conviction that when you are in a plane or car, you are moving and the land that you see go by must necessarily be at rest. Perhaps, under some circumstances it is more logical to think of yourself at rest and the ground as moving. *Or, perhaps it does not make any difference.* The principle of relativity allows us to take this last point of view.

Statement of the Principle of Relativity

Earlier we defined uniform motion as motion at constant speed in a straight line. And we mentioned that the principle of relativity said that you could not feel this uniform motion. Since it is not exactly clear what is meant by "feeling" uniform motion, a more precise statement of the principle of relativity is needed, a statement that can be tested by experiment. The following is the definition we will use in this text.

Imagine that you are in a capsule and you may have any equipment you wish inside the capsule. The principle of relativity states that there is no experiment you can perform that will allow you to tell whether or not the capsule is moving with uniform motion—motion in a straight line at constant speed.

In the above definition the capsule can use anything you want as an example—a jet plane, a car, or a room in a building. Generally, think of it as a sealed capsule like the jet plane where the moving picture is being shown and all of the window shades are shut. Of course you can look outside, and you may see things going by. But, as shown in Figure (1), seeing things outside go by does not prove that you and the capsule are moving. That cannot be used as evidence of your own uniform motion.

Think about what kind of experiments you might perform in the sealed capsule to detect your uniform motion. One experiment is to drop a coin on the floor. If you are at rest, the coin falls straight down. But if you are in a jet travelling 500 miles per hour and the flight is smooth, and you drop a coin, the coin still falls straight down. Dropping a coin does not distinguish between being at rest or moving at 500 miles per hour; this is one experiment that does not violate the principle of relativity.

There are many other experiments you can perform. You could use gyroscopes, electronic circuits, nuclear reactions, gravitational wave detectors, anything you want. The principle of relativity states that none of these will allow you to detect your uniform motion.

Exercise 2

Think about what you might put inside the capsule and what experiments you might perform to detect the motion of the capsule. Discuss your ideas with others and see if you can come up with some way of violating the principle of relativity.

Basic Law of Physics

We mentioned that one of the incentives for beginning the text with the principle of relativity is that it is an excellent example of a basic law of physics. It is simple and easy to state—there is no experiment that you can perform that allows you to detect your own uniform motion. Yet it is general—there is no experiment that can be done at any time, at any place, using anything, that can detect your uniform motion. And most important, it is completely subject to experimental test on an all-or-nothing basis. Just one verifiable experiment detecting one's own uniform motion, and the principle of relativity is no longer a basic law. It may become a useful approximation, but not a basic law.

Once a fundamental law like the principle of relativity is discovered or accepted, it has a profound effect on the way we think about things. In this case, if there is no way that we can detect our own uniform motion, then we might as well ignore our motion and always assume that we are at rest. Nature is usually easier to explain if we take the point of view that we are at rest and that other people and things are moving by. It is the principle of relativity that allows us to take this self-centered point of view.

It is a shock, a lot of excitement is generated, when what was accepted as a basic law of physics is discovered not to be exactly true. The discovery usually occurs in some obscure corner of science where no one thought to look before. And it will probably have little effect on most practical applications. But the failure of a basic law changes the way we think.

Suppose, for example, that it was discovered that the principle of relativity did not apply to the decay of an esoteric elementary particle created only in the gigantic particle accelerating machines physicists have recently built. This violation of the principle of relativity would have no practical effect on our daily lives, but it would have a profound psychological effect. We would then know that our uniform motion could be detected, and therefore on a fundamental basis we could no longer take the point of view that we are at rest and others are moving. There would be legitimate debates as to who was moving and who was at rest. We would search for a formulation of the laws of physics that made it intuitively clear who was moving and who was at rest.

This is almost what happened in 1860. In that year, James Clerk Maxwell summarized the laws of electricity and magnetism in four short equations. He then solved these equations to predict the existence of a wave of electric and magnetic force that should travel at a speed of approximately 3×10^8 meters per second. The predicted speed, which we will call c , could be determined from simple measurements of the behavior of an electric circuit.

Before Maxwell, no one had considered the possibility that electric and magnetic forces could combine in a wavelike structure that could travel through space. The first question Maxwell had to answer was what this wave was. Did it really exist? Or was it some spurious solution of his equations?

The clue was that the speed c of this wave was so fast that only light had a comparable speed. And more remarkably the known speed of light, and the speed c of his wave were very close—to within experimental error they were equal. As a consequence Maxwell proposed that he had discovered the theory of light, and that this wave of electric and magnetic force was light itself.

Maxwell's theory explained properties of light such as polarization, and made predictions like the existence of radio waves. Many predictions were soon verified, and within a few years there was little doubt that Maxwell had discovered the theory of light.

One problem with Maxwell's theory is that by measurements of the speed of light, it appears that one should be able to detect one's own uniform motion. In the next section we shall see why. This had two immediate consequences. One was a change in the view of nature to make it easy to see who was moving and who was not. The second was a series of experiments to see if the earth were moving or not.

In the resulting view of nature, all of space was filled with an invisible substance called *ether*. Light was pictured as a wave in the ether medium just as ocean waves are waves in the medium of water. The experiments, initiated by Michaelson and Morley, were designed to detect the motion of the earth by measuring how fast the earth was moving through the ether medium.

The problem with the ether theory was that all experiments designed to detect ether, or to detect motion through it, seemed to fail. The more clever the experiment, the more subtle the apparent reason for the failure. We will not engage in any further discussion of the ether theory, because ether still has never been detected. But we will take a serious look in the next section at how the measurement of the speed of a pulse of light should allow us to detect our own uniform motion. And then in the rest of the chapter we will discuss how a young physicist, working in a patent office in 1905, handled the problem.

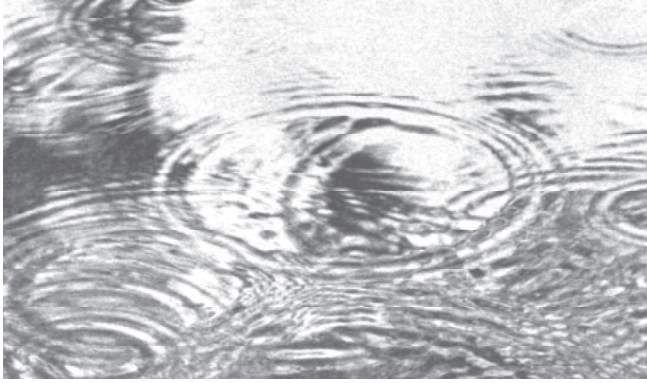


Figure 2
Rain drops creating circular waves on the surface of a puddle. (Courtesy Bill Jack Rodgers, Los Alamos Scientific Laboratory.)



Figure 3
This ocean wave traveled hundreds of miles from Hurricane Bertha to the Maine coast (July 31, 1990).

WAVE MOTION

We do not need to know the details of Maxwell’s theory to appreciate how one should be able to use the theory to violate the principle of relativity. All we need is an understanding of some of the basic properties of wave motion.

The most familiar examples of wave motion are the waves on the surface of water. We have seen the waves that spread out in circles when a stone is dropped in a pond, or rain hits a puddle in a sidewalk as shown in Figure (2). And most of us have seen the ocean waves destroying themselves as they crash into the beach. The larger ocean waves often originate at a storm far out to sea, and have traveled hundreds or even a thousand miles to reach you (see Figure (3)). The very largest ocean waves, created by earthquakes or exploding volcanos have been known to travel almost around the earth.

Although you cannot see them, sound waves are a more familiar form of wave motion. Sound moving through air, waves moving over water, and light, all have certain common features and ways of behaving which we classify as *wave motion*. In later chapters we will study the subject of wave motion in considerable detail. For now we will limit our discussion to a few of the features we need to understand the impact of Maxwell’s theory.

Two examples of wave motion that are easy to study are a wave pulse traveling down a rope as indicated in Figure (4) or down a stretched Slinky® (the toy coil that ‘climbs’ down stairs) as shown in Figure (5). The advantage of using a stretched Slinky is that the waves travel so slowly that you can study them as they move. It turns out that the speed of a wave pulse depends upon the medium along which, or through which, it is traveling. For example, the speed of a wave pulse along a rope or Slinky is given by the formula

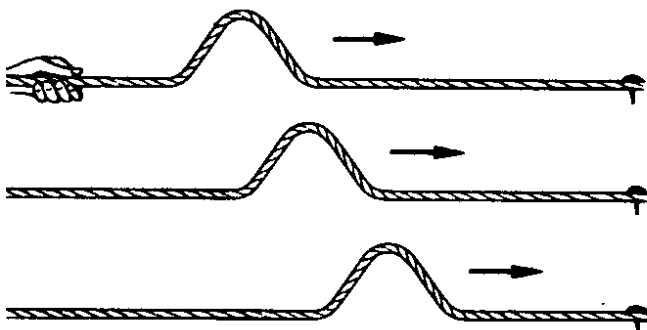


Figure 4
Wave pulse traveling along a rope.

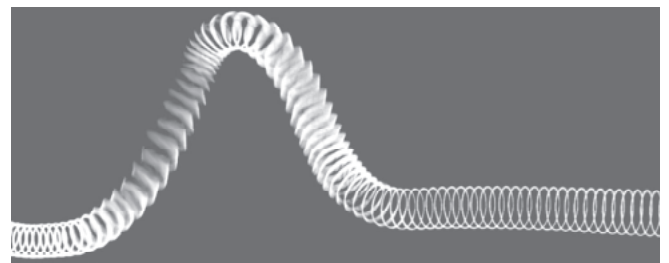


Figure 5
Wave pulse traveling along a Slinky.

$$\text{Speed of wave pulse} = \sqrt{\frac{\tau}{\mu}} \quad (1)$$

where τ is the tension in the rope or Slinky, and μ the mass per unit length. Do not worry about precise definitions of tension or mass, the important point is that there is a formula for the speed of the wave pulse, a formula that depends only on the properties of the medium along which the pulse is moving.

The speed does not depend upon the shape of the pulse or how the pulse was created. For example, the Slinky pulse travels much more slowly than the pulse on the rope because the suspended Slinky has very little tension τ . We can slow the Slinky wave down even more by hanging crumpled pieces of lead on each end of the coils of the Slinky to increase its mass per unit length μ .

Another kind of wave we can create in the Slinky is the so called *compressional wave* shown in Figure (6). Here the end of the Slinky was pulled back and released, giving a moving pulse of compressed coils. The formula for the speed of the compression wave is still given by Equation (1), if we interpret τ as the stiffness (*Youngs modulus*) of the suspended Slinky.

If we use a loudspeaker to produce a compressional pulse in air, we get a sound wave that travels out from the loudspeaker at the speed of sound. The formula for the speed of a sound wave is

$$\text{Speed of sound} = \sqrt{\frac{B}{\rho}} \quad (2)$$

where B is the *bulk modulus* which can be thought of as the rigidness of the material, and the mass per unit length μ is replaced by the mass per unit volume ρ .

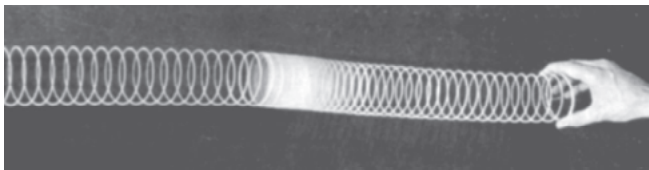


Figure 6
To create a compressional wave on a suspended Slinky, pull the end back a bit and let go.

A substance like air, which is relatively compressible, has a small rigidity B , while substances like steel and granite are very rigid and have large values of B . As a result sound travels much faster in steel and granite than in air. For air at room temperature and one atmosphere of pressure, the speed of sound is 343 meters or 1125 feet per second. Sound travels about 20 times faster in steel and granite. Again the important point is that the speed of a wave depends on the properties of the medium through which it is moving, and not on the shape of the wave or the way it was produced.

Measurement of the Speed of Waves

If you want to know how fast your car is traveling you look at the speedometer. Some unknown machinery in the car makes the needle of the speedometer point at the correct speed. Since the wave pulses we are discussing do not have speedometers, we have to carry out a series of measurements in order to determine their speed. In this section we wish to discuss precisely how the measurements can be made using meter sticks and clocks so that there will be no ambiguity, no doubt about precisely what we mean when we talk about the speed of a wave pulse. We will use the Slinky wave pulse as our example, because the wave travels slowly enough to actually carry out these measurements in a classroom demonstration.

The first experiment, shown in Figure (7), involves two students and the instructor. One student stands at the end of the stretched Slinky and releases a wave pulse like that shown in Figure (6). The instructor holds a meter stick up beside the Slinky as shown. The other student has a stopwatch and measures the length of time it takes the pulse to travel from the front to the back of the stick. (She presses the button once when the pulse reaches the front of the meter stick, presses it again when the pulse gets to the back, and reads the elapsed time T .) The speed of the pulse is then defined to be

$$\text{Speed of Slinky pulse} = \frac{1 \text{ meter}}{T \text{ seconds}} \quad (3)$$

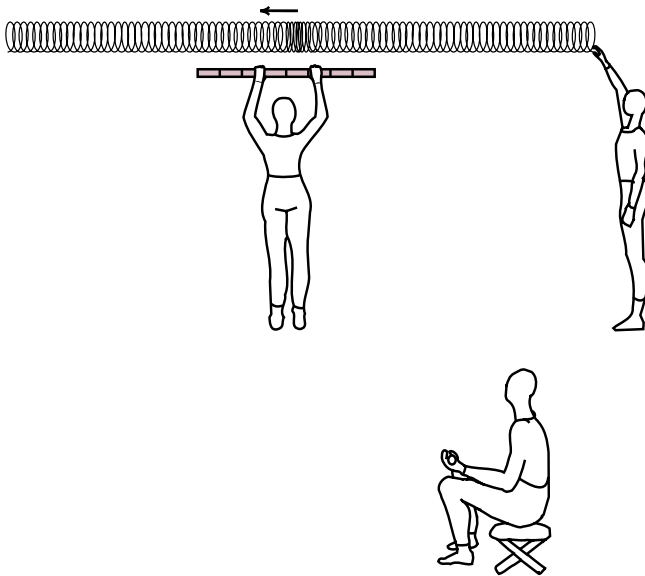


Figure 7
Experiment to measure the speed of a wave pulse on a suspended Slinky. Here the instructor holds the meter stick at rest.

Later in the course, when we have discussed ways of measuring tension τ and mass per unit length μ , we can compare the experimental result we get from Equation (3) with the theoretically predicted result of Equation (1). With a little practice using the stopwatch, it is not difficult to get reasonable agreement between theory and experiment .

In our second experiment, shown in Figure (8a) everything is the same except that the instructor has been replaced by a student, let us say it is Bill, holding the meter stick and running toward the student who releases the wave pulse. Again the second student measures the length of time it takes the pulse to travel from the front to the back of the meter stick. Let us call

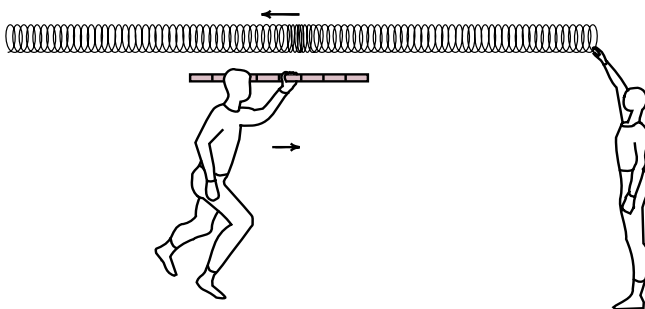


Figure 8a
Bill runs toward the source of the pulse while measuring its speed.

this the time T_1 . This time T_1 is less than T because Bill and the meter stick are moving toward the pulse.

To Bill, the pulse passes his one meter long stick in a time T_1 , therefore the speed of the pulse past him is

$$v_1 = \frac{1 \text{ meter}}{T_1 \text{ seconds}} = \text{speed of pulse relative to Bill} \quad (4a)$$

Bill should also have carried the stopwatch so that v_1 would truly represent his measurement of the speed of the pulse. But it is too awkward to hold the meter stick, and run and observe when the pulse is passing the ends of the stick.

The speed v_1 measured by Bill is not the same as the speed v measured by the instructor in Figure (7). v_1 is greater than v because Bill is moving toward the wave pulse. This is not surprising: if you are on a freeway and everyone is traveling at a speed $v = 55$ miles per hour, the oncoming traffic in the opposite lane is traveling past you at a speed of 110 miles per hour because you are moving toward them.

In Figure (8b) we again have the same situation as in Figure (7) except that Bill is now replaced by Joan who is running away from the student who releases the pulse. Joan is moving in the same direction as the pulse and it takes a longer time T_2 for the pulse to pass her. (Assume that Joan is not running faster than the pulse.) The speed of the pulse relative to Joan is

$$v_2 = \frac{1 \text{ meter}}{T_2 \text{ seconds}} = \text{speed of pulse relative to Joan} \quad (4b)$$

Joan's speed v_2 will be considerably less than the speed v observed by the instructor.

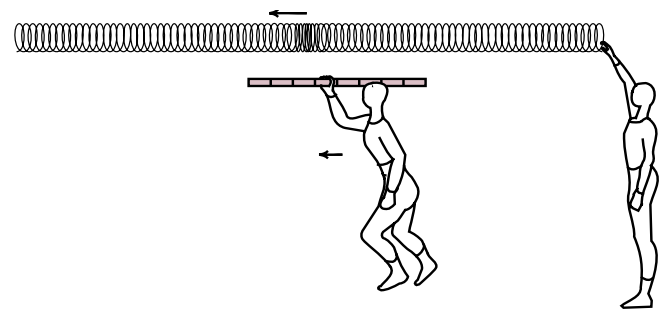


Figure 8b
Joan runs away from the source of the pulse while measuring its speed.

In these three experiments, the instructor is special (wouldn't you know it). Only the instructor measures the speed v predicted by theory, only for the instructor is the speed given by $v = \sqrt{\tau/\mu}$. Both the students Bill and Joan observe different speeds, one larger and one smaller than the theoretical value.

What is special about the instructor? In this case the instructor gets the predicted answer because she is at rest relative to the Slinky. If we hadn't seen the experiment, but just looked at the answers, we could tell that the instructor was at rest because her result agreed with the predicted speed of a Slinky wave. Bill got too high a value because he was moving toward the pulse; Joan, too low a value because she was moving in the same direction.

The above set of experiments is not strikingly profound. In a sense, we have developed a new and rather cumbersome way to tell who is not moving relative to the Slinky. But the same procedures can be applied to a series of experiments that gives more interesting results. In the new series of experiments, we will use a pulse of light rather than a wave pulse on a Slinky.

Since the equipment is not likely to be available among the standard set of demonstration apparatus, and since it will be difficult to run at speeds comparable to the speed of light we will do this as a thought experiment. We will imagine that we can measure the time it takes a light pulse to go from the front to the back of a meter stick. We will imagine the kind of results we expect to

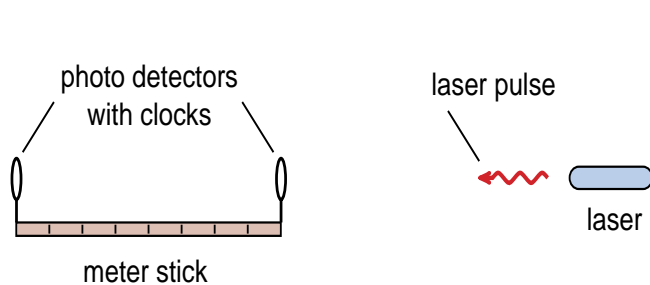


Figure 9

Apparatus for the thought experiment. Now we wish to measure the speed of a laser wave pulse, rather than the speed of a Slinky wave pulse. The photo detectors are used to measure the length of time the laser pulse takes to pass by the meter stick.

get, and then see what the consequences would be if we actually got those results.

The apparatus for our new thought experiment is shown in Figure (9). We have a laser which can produce a very short pulse of light – only a few millimeters long. The meter stick now has photo detectors and clocks mounted on each end, so that we can accurately record the times at which the pulse of light passed each end. These clocks were synchronized, so the time difference is the length of time T it takes the pulse of light to pass the meter stick.

Before the experiment, the instructor gives a short lecture to the class. She points out that according to Maxwell's theory of light, a light wave should travel at a speed c given by the formula

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad (5)$$

where μ_0 and ϵ_0 are constants in the theory of electricity. She says that later on in the year, the students will perform an experiment in which they measure the value of the product $\mu_0 \epsilon_0$. This experiment involves measuring the size of coils of wire and plates of aluminum, and timing the oscillation of an electric current sloshing back and forth between the plates and the coil. The important point is that these measurements do not involve light. It is analogous to the Slinky where the predicted speed $\sqrt{\tau/\mu}$ of a Slinky wave involved measurements of the stiffness τ and mass per unit length μ , and had nothing to do with observations of a Slinky wave pulse.

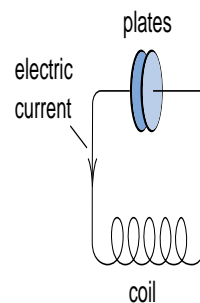


Figure 10a
Plates and coil for measuring the experimental value of $\mu_0 \epsilon_0$.



Figure 10b
The plates and coil we use in the laboratory.

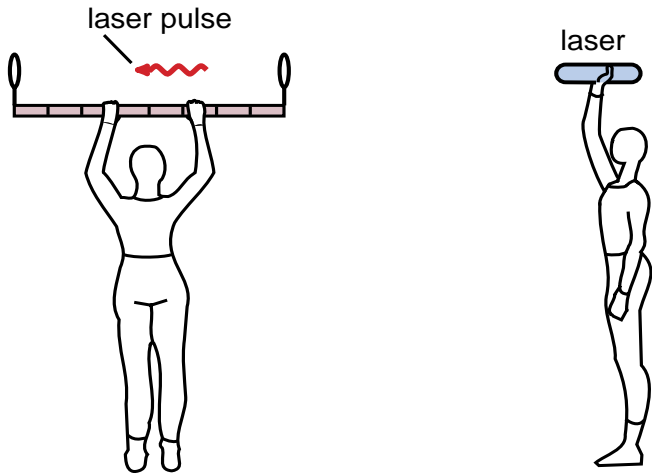


Figure 11
Experiment to measure the speed of a light wave pulse from a laser. Here the instructor holds the meter stick at rest.

Although she is giving out the answer to the lab experiment, she points out that the value of c from these measurements is

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ meters/second} \quad (6)$$

which is a well-known but uncomfortably large and hard to remember number. However, she points out, 3×10^8 meters is almost exactly one billion (10^9) feet. If you measure time, not in seconds, but in billionths of a second, or *nanoseconds*, where

$$1 \text{ nanosecond} \equiv 10^{-9} \text{ seconds} \quad (7)$$

then since light travels only one foot in a nanosecond, the speed of light is simply

$$c = 3 \times 10^8 \frac{\text{meter}}{\text{sec}} = 10^9 \frac{\text{feet}}{\text{sec}} \quad (8)$$

$$c = \frac{1 \text{ foot}}{10^{-9} \text{ sec}}$$

$$c = 1 \frac{\text{foot}}{\text{nanosecond}}$$

She says that because this is such an easy number to remember, she will use it throughout the rest of the course.

The lecture on Maxwell's theory being over, the instructor starts in on the thought experiment. In the first run she stands still, holding the meter stick, and the student with the laser emits a pulse of light as shown in Figure (11). The pulse passes the 3.28 foot length of the meter stick in an elapsed time of 3.28 nanoseconds, for a measured speed

$$v(\text{light pulse}) = \frac{3.28 \text{ feet}}{3.28 \text{ nanoseconds}} = 1 \frac{\text{foot}}{\text{nanosecond}} \quad (9)$$

The teacher notes, with a bit of complacency, that she got the predicted speed of 1 foot/nanosecond. Again, the instructor is special.

Then the instructor invites Bill to hold the meter stick and run toward the laser as shown in Figure (12a). Since this is a thought experiment, she asks Bill to run at nearly the speed of light, so that the time should be cut in half and Bill should see light pass him at nearly a speed of $2c$.

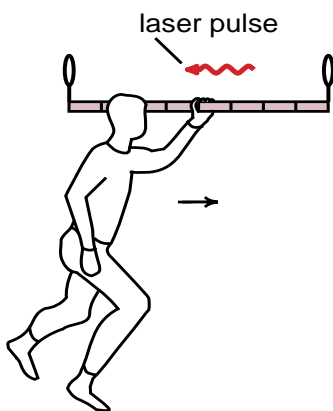


Figure 12a
Bill runs toward the source of the pulse while measuring its speed.

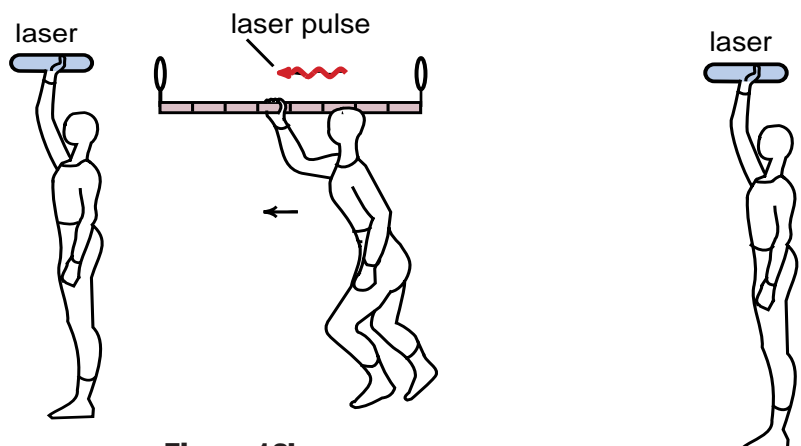


Figure 12b
Joan runs away from the source of the pulse while measuring its speed.

Then she invites Joan to hold the meter stick and run at about half the speed of light in the other direction as shown in Figure (12b). One would expect that the light would take twice as long to pass Joan as it did the instructor and that Joan should obtain a value of about $c/2$ for the speed of light.

Suppose it turned out this way. Suppose that the instructor got the predicted answer 1 foot/nanosecond, while Bill who is running toward the pulse got a higher value and Joan, running with the pulse got a lower value. Just as in our Slinky pulse experiment we could say that the instructor was at rest while both Bill and Joan were moving.

But, moving relative to what? In the Slinky experiment, the instructor was at rest relative to the Slinky – the medium through which a Slinky wave moves. Light pulses travel through empty space. Light comes to us from stars 10 billion light years away, almost across the entire universe. The medium through which light moves is empty space.

If the experiment came out the way we described, the instructor would have determined that she was at rest relative to empty space, while Bill and Joan would have determined that they were moving. **They would have violated the principle of relativity**, which says that you cannot detect your own motion relative to empty space.

The alert student might argue that the pulses of light come out of the laser like bullets from a gun at a definite muzzle velocity, and that all the instructor, Bill and Joan are doing is measuring their speed relative to the laser. Experiments have carefully demonstrated that the speed of a pulse of light depends in no way on the motion of the emitter just as the Slinky pulse depended in no way on how the student started the pulse. Maxwell's theory predicts that light is a wave, and many experiments have verified the wave nature of light, including the fact that its speed does not depend on how it was emitted.

From the logical simplicity of the above thought experiment, from the ease with which we should be able to violate the principle of relativity (if we could accurately measure the speed of a pulse of light passing us), it is not surprising that after Maxwell developed this theory of light, physicists did not take the principle of relativity seriously, at least for the next 45 years.

Michaelson-Morley Experiment

The period from 1860 to 1905 saw a number of attempts to detect one's own or the earth's motion through space by measuring the speed of pulses of light. Actually it was easier and far more accurate to compare the speeds of light traveling in different directions. If you were moving forward through space (like Bill in our thought experiment), you should see light coming from in front of you traveling faster than light from behind or even from the side.

Michaelson and Morley used a device called a Michaelson interferometer which compared the speeds of pulses of light traveling at right angles to each other. A detailed analysis of their device is not hard, just a bit lengthy. But the result was that the device should be able to detect small differences in speeds, small enough differences so that the motion of the earth through space should be observable -- even the motion caused by the earth orbiting the sun.

At this point we can summarize volumes of the history of science by pointing out that no experiment using the Michaelson interferometer, or any device based on measuring or comparing the speed of light pulses, ever succeeded in detecting the motion of the earth.

Exercise 3

Units of time we will often use in this course are the millisecond, the microsecond, and the nanosecond, where

$$1 \text{ millisecond} = 10^{-3} \text{ seconds (one thousandth)}$$

$$1 \text{ microsecond} = 10^{-6} \text{ seconds (one millionth)}$$

$$1 \text{ nanosecond} = 10^{-9} \text{ seconds (one billionth)}$$

How many feet does light travel in

- one millisecond (1ms)?
 - one microsecond (1 μ s)?
 - one nanosecond (1ns)?
-

EINSTEIN'S PRINCIPLE OF RELATIVITY

In 1905 Albert Einstein provided a new perspective on the problems we have been discussing. He was apparently unaware of the Michaelson-Morley experiments. Instead, Einstein was familiar with Maxwell's equations for electricity and magnetism, and noted that these equations had a far simpler form if you took the point of view that you are at rest. He suggested that these equations took this simple form, not just for some privileged observer, but for everybody. If the principle of relativity were correct after all, then everyone, no matter how they were moving, could take the point of view that they were at rest and use the simple form of Maxwell's equations.

How did Einstein deal with measurements of the speed of light? We have seen that if someone, like Bill in our thought experiment, detects a pulse of light coming at them at a speed faster than $c = 1 \text{ foot/nanosecond}$, then that person could conclude that they themselves were moving in the direction from which the light was coming. They would have thereby violated the principle of relativity.

Einstein's solution to that problem was simple. He noted that any measurement of the speed of a pulse of light that gave an answer different from $c = 1 \text{ foot/nanosecond}$ could be used to violate the principle of relativity. Thus if the principle of relativity were correct, *all measurements of the speed of light must give the answer c .*

Let us put this in terms of our thought experiment. Suppose the instructor observed that the light pulse passed the 3.24 foot long meter stick in precisely 3.24 nanoseconds. And suppose that Bill, moving at nearly the speed of light toward the laser, also observed that the light took 3.24 nanoseconds to pass by his meter stick. And suppose that Joan, moving away from the laser at half the speed of light, also observed that the pulse of light took 3.24 nanoseconds to pass by her meter stick. If the instructor, Bill and Joan all got precisely the same answer for the speed of light, then none of their results could be used to prove that one was at rest and the others moving. Since their answer of 3.24 feet in 3.24 nanoseconds or 1 foot/nanosecond is in agreement with the predicted value $c = 1/\sqrt{\mu_0\epsilon_0}$

from Maxwell's theory, they could all safely assume that they were at rest. At the very least, their measurements of the speed of the light pulse could not be used to detect their own motion.

As we said, the idea is simple. You always get the answer c whenever you measure the speed of a light pulse moving past you. But the idea is horrendous. Einstein went against more than 200 years of physics and centuries of observation with this suggestion.

Suppose, for example, we heard about a freeway where all cars traveled at precisely 55 miles per hour – **no exceptions**. Hearing about this freeway, our three people in the thought experiment decide to test the rule. The instructor sets up measuring equipment in the median strip and observes that the rule is correct. Cars in the north bound lane travel north at 55 miles per hour, and cars in the south bound lane go south at 55 miles per hour.

For his part of the experiment, Bill gets into one of the north bound cars. Since Bill knows about the principle of relativity he takes the point of view that he is at rest. If the 55 miles per hour speed is truly a fundamental law, then he, who is at rest, should see the south bound cars pass at 55 miles per hour.

Likewise, Joan, who is in a south bound car, can take the point of view that she is at rest. She knows that if the 55 miles per hour speed limit is a fundamental law, then north bound cars must pass her at precisely 55 miles per hour. If the instructor, Bill and Joan all observe that every car on the freeway always passes them at the same speed of 55 miles per hour, then none of them can use this observation to detect their own motion.

Freeways do not work that way. Bill will see south bound cars passing him at 110 miles per hour. And Joan will see north bound cars passing at 110 miles per hour. From these observations Bill and Joan will conclude that in fact they are moving – at least relative to the freeway.

Measurements of the speed of a pulse of light differ, however, in two significant ways from measurements of the speed of a car on a freeway. First of all, light moves through empty space, not relative to anything.

Secondly, light moves at enormous speeds, speeds that lie completely outside the realm of common experience. Perhaps, just perhaps, the rules we have learned so well from common experience, do not apply to this realm. The great discoveries in physics often came when we look in some new realm on the very large scale, or the very small scale, or in this case on the scale of very large, unfamiliar speeds.

The Special Theory of Relativity

Einstein developed his special theory of relativity from two assumptions:

- 1) *The principle of relativity is correct.*
- 2) *Maxwell's theory of light is correct.*

As we have seen, the only way Maxwell's theory of light can be correct and not violate the principle of relativity, is that every observer who measures the speed of light, must get the predicted answer $c = 1/\sqrt{\mu_0\epsilon_0} = 1 \text{ foot/nanosecond}$. Temporarily we will use this as the statement of Einstein's second postulate:

- 2a) *Everyone, no matter how he or she is moving, must observe that light passes them at precisely the speed c .*

Postulates (1) and (2a) salvage both the principle of relativity and Maxwell's theory, but what else do they predict? We have seen that measurements of the speed of a pulse of light do not behave in the same way as measurements of the speed of cars on a freeway. Something peculiar seems to be happening at speeds near the speed of light. What are these peculiar things? How do we find out?

To determine the consequences of his two postulates, Einstein borrowed a technique from Galileo and used a series of thought experiments. Einstein did this so clearly, explained the consequences so well in his 1905 paper, that we will follow essentially the same line of reasoning. The main difference is that Einstein made a number of strange predictions that in 1905 were hard to believe. But these predictions were not only verified, they became the cornerstone of much of 20th century physics. We will be able to cite numerous tests of all the predictions.

Moving Clocks

Our first thought experiment for Einstein's special relativity will deal with the behavior of clocks. We saw that the measurement of the speed of a pulse of light required a timing device, and perhaps the peculiar results can be explained by the peculiar behavior of the timing device.

Also the peculiar behavior seems to happen at high speeds near the speed of light, not down at freeway speeds. Thus the question we would like to ask is what happens to a clock that is moving at a high speed, near the speed of light?

That is a tough question. There are many kinds of clocks, ranging from hour glasses dripping sand, to the popular digital quartz watches, to the atomic clocks used by the National Bureau of Standards. The oldest clock, from which we derive our unit of time, is the motion of the earth on its axis each 24 hours. We have both the problem of deciding which kind of clock we wish to consider moving at high speeds, and then figure out how that clock behaves.

The secret of working with thought experiments is to keep everything as simple as possible and do not try to do too much at once. If we want to understand what happens to a moving clock, we should start with the simplest clock we can find. If we cannot understand that one, we will imagine an even simpler one.

A clock that is fairly easy to understand is the old grandfather's clock shown in Figure (13), where the timing device is the swinging pendulum. There are also wheels, gears, and hands, but these merely count swings of the pendulum. The pendulum itself is what is important. If you shorten the pendulum it swings faster and the hands go around faster.

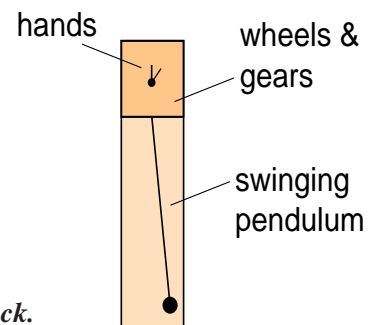


Figure 13
Grandfather's clock.

We could ask what we would see if we observed a grandfather's clock moving past us at a high speed, near the speed of light. The answer is likely to be "I don't know". The grandfather's clock, with its swinging pendulum mechanism, is still too complicated.

A simpler timing device was considered by Einstein, namely a bouncing pulse of light. Suppose, we took the grandfather's clock of Figure (13), and replaced the pendulum by two mirrors and a pulse of light as shown in Figure (14). Space the mirrors 1 foot apart so that the pulse of light will take precisely one nanosecond to bounce either up or down. Leave the rest of the machinery of the grandfather's clock more or less intact. In other words have the wheels and gears now count bounces of the pulse of light rather than swings of the pendulum. And recalibrate the face of the clock so that for each bounce, the hand advances one nanosecond. (The marvelous thing about thought experiments is that you can get away with this. You do not have to worry about technical feasibility, only logical consistency.)

The advantage of replacing the pendulum with a bouncing light pulse is that, so far, the only thing whose behavior we understand when moving at nearly the speed of light is light itself. We know that light always moves at the speed c in all circumstances, to any observer. If we use a bouncing light pulse as a timing device, and can figure out how the pulse behaves, then we can figure out how the clock behaves.

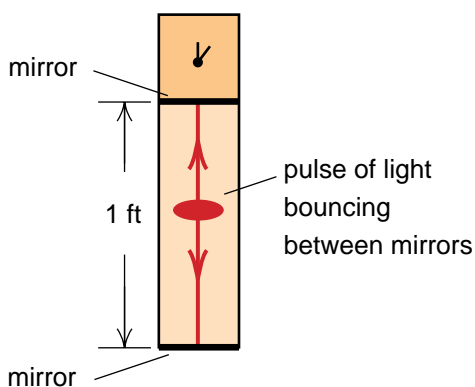


Figure 14
Light pulse clock. We can construct a clock by having a pulse of light bounce between two mirrors. If the mirrors are one foot apart, then the time between bounces will be one nanosecond. The face of the clock displays the number of bounces.

For our thought experiment it is convenient to construct two identical light pulse clocks as shown in Figure (15). We wish to take great care that they are identical, or at least that they run at precisely the same rate. Once they are finished, we adjust them so that the pulses bounce up and down together for weeks on end.

Now we get to the really hypothetical part of our thought experiment. We give one of the clocks to an astronaut, and we keep the other for reference. The astronaut is instructed to carefully pack his clock, accelerate up to nearly the speed of light, unpack his clock, and go by us at a constant speed so that we can compare our reference clock to his moving clock.

Before we describe what we see, let us take a look at a brief summary of the astronaut's log book of the trip. The astronaut writes, "I carefully packed the light pulse clock because I did not want it damaged during the accelerations. My ship can maintain an acceleration of 5gs, and even then it took about a month to get up to our final speed of just over half the speed of light."

"Once the accelerations were over and I was coasting, I took the light pulse clock out of its packing and set it up beside the window, so that the class could see the clock as it went by. Before the trip I was worried that I might have some trouble getting the light pulse into the clock, but it was no problem at all. I couldn't even tell that I was moving! The light pulse went in and the clock started ticking just the way it did back in the lab, before we started the trip."

"It was not long after I started coasting, that the class went by. After that, I packed everything up again, decelerated, and returned to earth."

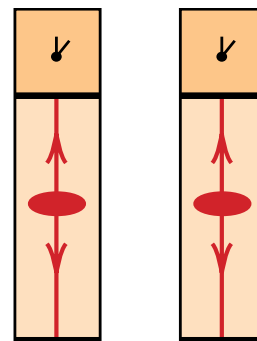
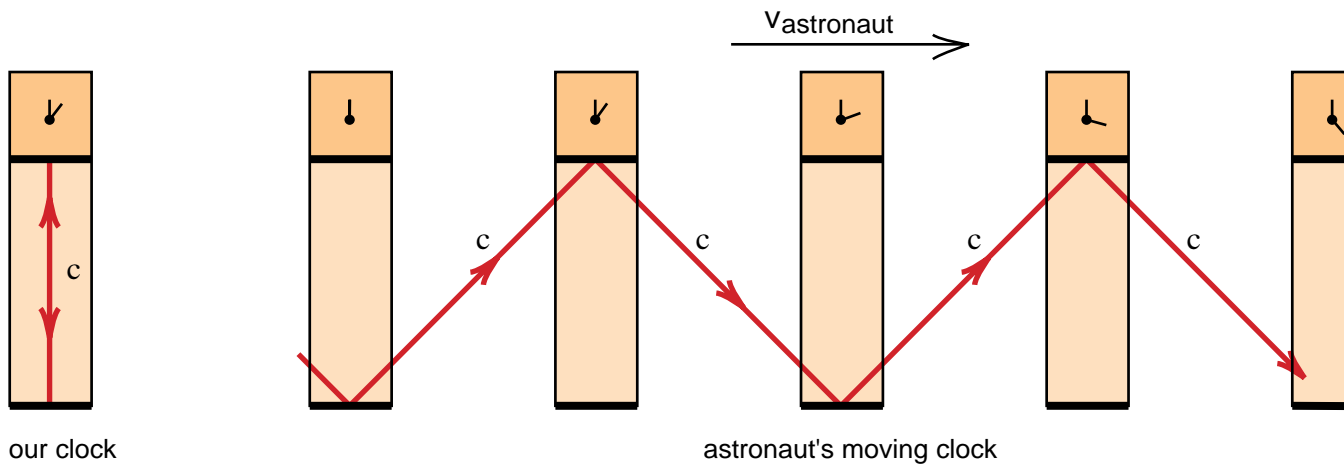


Figure 15
Two identical light pulse clocks.

**Figure 16**

In order to stay in the astronaut's moving clock, the light pulse must follow a longer, saw-tooth, path.

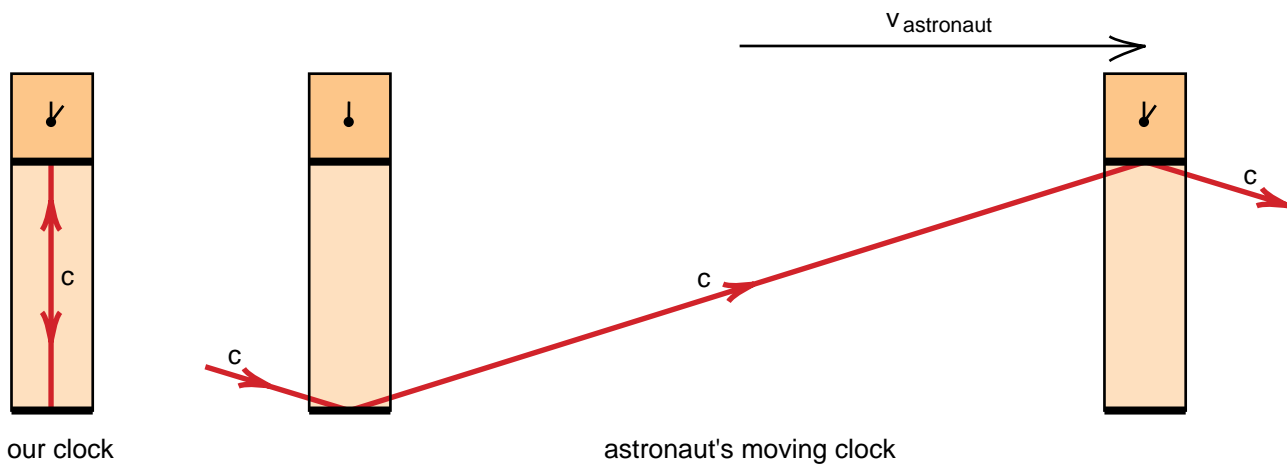
What we saw as the astronaut went by is illustrated in the sketch of Figure (16). On the left is our reference clock, on the right the astronaut's clock moving by. You will recall that the astronaut had no difficulty getting the light pulse to bounce, and as a result we saw his clock go by with the pulse bouncing inside.

For his pulse to stay in his clock, his pulse had to travel along the saw-tooth path shown in Figure (16). The saw-tooth path is longer than the up and down path taken by the pulse in our reference clock. His pulse had to travel farther than our pulse to tick off one nanosecond.

Here is what is peculiar. If Einstein's postulate is right, if the speed of a pulse of light is always c under any circumstances, then our pulse bouncing up and down,

and the astronaut's pulse traveling along the saw tooth path are both traveling at the same speed c . Since the astronaut's pulse travels farther, the astronaut's clock must take longer to tick off a nanosecond. ***The astronaut's clock must be running slower!***

Because there are no budget constraints in a thought experiment, we are able to get a better understanding of how the astronaut's clock was behaving by having the astronaut repeat the trip, this time going faster, about $.95c$. What we saw is shown in Figure (17). The astronaut's clock is moving so fast that the saw tooth path is stretched way out. The astronaut's pulse takes a long time to climb from the bottom to the top mirror in his clock, his nanoseconds take a long time, and his clock runs very slowly.

**Figure 17**

When the astronaut goes faster, his light pulse has to go farther in order to register a bounce. Since the speed of light does not change, it takes longer for one bounce to register, and the astronaut's moving clock runs slower.

It does not take too much imagination to see that if the astronaut came by at the speed of light c , the light pulse, also traveling at a speed c , would have to go straight ahead just to stay in the clock. It would never be able to get from the bottom to the top mirror, and his clock would never tick off a nanosecond. His clock would stop!

Exercise 4

Discuss what the astronaut should have seen when the class of students went by. In particular, draw the astronaut's version of Figure (16) and describe the situation from the astronaut's point of view.

It is not particularly difficult to calculate the amount by which the astronaut's clock runs slow. All that is required is the Pythagorean theorem. In Figure (18), on the left, we show the path of the light pulse in our reference clock, and on the right the path in the astronaut's moving clock. Let T be the length of time it takes our pulse to go from the bottom to the top mirror, and T' the longer time light takes to travel along the diagonal line from his bottom mirror to his top mirror. We can think of T as the length of one of our nanoseconds, and T' as the length of one of the astronaut's longer nanoseconds.

The distance an object, moving at a speed v , travels in a time T , is vT . (If you go 30 miles per hour for 3 hours, you travel 90 miles.) Thus, in Figure (18), the distance our light pulse travels in going from the bottom to the top mirror is cT as shown. The astronaut's light pulse, which takes a time T' to travel the diagonal path, must have gone a distance cT' as shown.

During the time T' , while the astronaut's light pulse is going along the diagonal path, the astronaut's clock, which is traveling at a speed v , moves forward a

distance vT' as shown. This gives us a right triangle whose base is vT' , whose hypotenuse is cT' , and whose height, determined from our clock, is cT . According to the Pythagorean theorem, these sides are related by

$$(cT')^2 = (vT')^2 + (cT)^2 \tag{10}$$

Carrying out the squares, and collecting the terms with T' on one side, we get

$$c^2T'^2 - v^2T'^2 = (c^2 - v^2)T'^2 = c^2T^2$$

$$T'^2 = \frac{c^2T^2}{c^2 - v^2} = \frac{c^2T^2 \times (1/c^2)}{(c^2 - v^2) \times (1/c^2)} = \frac{T^2}{1 - v^2/c^2}$$

Taking the square root of both sides gives

$$T' = \frac{T}{\sqrt{1 - v^2/c^2}} \tag{11}$$

Equation (11) gives a precise relationship between the length of our nanosecond T and the astronaut's longer nanosecond T' . We see that the astronaut's basic time unit T' is longer than our basic time unit T by a factor $1/\sqrt{1 - v^2/c^2}$.

The factor $1/\sqrt{1 - v^2/c^2}$ appears in a number of calculations involving Einstein's special theory of relativity. As a result, it is essential to develop an intuitive feeling for this number. Let us consider several examples to begin to build this intuition. If $v = 0$, then

$$T' = \frac{T}{\sqrt{1 - v^2/c^2}} = \frac{T}{\sqrt{1 - 0}}$$

$$T' = \frac{T}{1} = T \quad (v = 0) \tag{12}$$

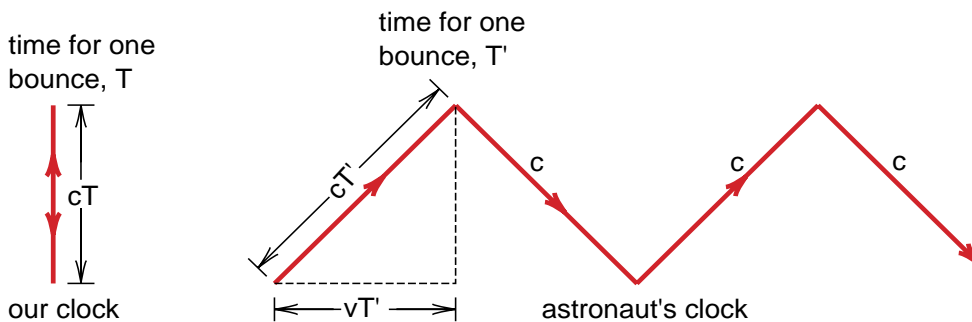


Figure 18
In our clock, the light pulse travels a distance cT in one bounce. In the astronaut's clock, the pulse travels a distance cT' while the clock moves forward a distance vT' during one bounce.

and we see that a clock at rest keeps the same time as ours. If the astronaut goes by at one tenth the speed of light, $v = .1c$, and we get

$$T' = \frac{T}{\sqrt{1 - (.1c)^2/c^2}} = \frac{T}{\sqrt{1 - .01}}$$

$$T' = \frac{T}{\sqrt{.99}} = 1.005T \quad (v = c/10) \quad (13)$$

In this case the astronaut's seconds lengthen only by a factor 1.005 which represents only a .5% increase. If the astronaut's speed is increased to half the speed of light, we get

$$T' = \frac{T}{\sqrt{1 - (.5c)^2/c^2}} = \frac{T}{\sqrt{1 - .25}}$$

$$T' = \frac{T}{\sqrt{.75}} = 1.15T \quad (v = c/2) \quad (14)$$

Now we are getting a 15% increase in the length of the astronaut's seconds.

When we work with atomic or subatomic particles, it is not difficult to accelerate these particles to speeds close to the speed of light. Shortly we will consider a particle called a muon, that is traveling at a speed $v = .994c$. For this particle we have

$$T' = \frac{T}{\sqrt{1 - (.994c)^2/c^2}} = \frac{T}{\sqrt{1 - .988}}$$

$$T' = \frac{T}{\sqrt{.012}} = 9T \quad (v = .994c) \quad (15)$$

Here we are beginning to see some large effects. If the astronaut were traveling this fast, his seconds would be 9 times longer than ours, his clock would be running only 1/9th as fast.

If we go all the way to $v = c$, Equation (11) gives

$$T' = \frac{T}{\sqrt{1 - c^2/c^2}} = \frac{T}{\sqrt{1 - 1}}$$

$$T' = \frac{T}{\sqrt{0}} = \infty \quad (v = c) \quad (16)$$

In this case, the astronaut's seconds would be infinitely long and the astronaut's clock would stop. This agrees with our earlier observation that if the astronaut went by at the speed of light, the light pulse in his clock would have to go straight ahead just to stay in the clock. It would not have time to move up or down, and therefore not be able to tick off any seconds.

So far we have been able to use a pocket calculator to evaluate $1/\sqrt{1 - v^2/c^2}$. But if the astronaut were flying in a commercial jet plane at a speed of 500 miles per hour, you have problems because $1/\sqrt{1 - v^2/c^2}$ is so close to 1 that the calculator cannot tell the difference. In a little while we will show you how to do such calculations, but for now we will just state the answer.

$$\frac{1}{\sqrt{1 - v^2/c^2}} = 1 + 2.7 \times 10^{-13} \text{ for a speed of } 500 \text{ mi/hr} \quad (17)$$

To put this result in perspective suppose the astronaut flew on the jet for what we thought was a time $T = 1$ hour or 3600 seconds. The astronaut's light pulse clock would show a longer time T' given by

$$T' = \frac{T}{\sqrt{1 - v^2/c^2}}$$

$$= (1 + 2.7 \times 10^{-13}) \times (3600 \text{ seconds})$$

$$= 1 \text{ hour} + .97 \times 10^{-9} \text{ seconds}$$

Since $.97 \times 10^{-9}$ seconds is close to a nanosecond, we can write

$$T' \approx 1 \text{ hour} + 1 \text{ nanosecond} \quad (18)$$

The astronaut's clock takes 1 hour plus 1 nanosecond to move its hand forward 1 hour. We would say that his light pulse clock is losing a nanosecond per hour.

Students have a tendency to memorize formulas, and Equation (11), $T' = T/\sqrt{1 - v^2/c^2}$ looks like a good candidate. **But don't!** If you memorize this formula, you will mix up T' and T , forgetting which seconds belong to whom. **There is a much easier way to always get the right answer.**

For any speed v less than or equal to c (which is all we will need to consider) the quantity $\sqrt{1 - v^2/c^2}$ is always a number less than or equal to 1, and $1/\sqrt{1 - v^2/c^2}$ is always greater than or equal to 1. For the examples we have considered so far, we have

Table 1

v	$\sqrt{1 - v^2/c^2}$	$1/\sqrt{1 - v^2/c^2}$
0	1	1
500 mi/hr	$1 - 2.7 \times 10^{-13}$	$1 + 2.7 \times 10^{-13}$
$c/10$.995	1.005
$c/2$.87	1.15
.994 c	1/9	9
c	0	∞

You also know intuitively that for the moving light pulse clock, the light pulse travels a longer path, and therefore the moving clock's seconds are longer.

If you remember that $\sqrt{1 - v^2/c^2}$ appears somewhere in the formula, all you have to do is ask yourself what to do with a number less than one to make the answer bigger; clearly, you have to divide by it.

As an example of this way of reasoning, note that if a moving clock's seconds are longer, then the rate of the clock is slower. The number of ticks per unit time is less. If we want to talk about the rate of a moving clock, do we multiply or divide by $\sqrt{1 - v^2/c^2}$? To get a reduced rate, we multiply by $\sqrt{1 - v^2/c^2}$ since that number is always less than one. Thus we can say that

the rate of the moving clock is reduced by a factor $\sqrt{1 - v^2/c^2}$.

The factor $\sqrt{1 - v^2/c^2}$ will appear numerous times throughout the text. But in every case you should have an intuitive idea of whether the quantity under consideration should increase or decrease. If it increases, divide by the $\sqrt{1 - v^2/c^2}$, and if it decreases, multiply by $\sqrt{1 - v^2/c^2}$. This approach gives the right answer, reduces memorization, and eliminates obscure notation like T' and T .

Other Clocks

So far we have an interesting but limited result. We have predicted that if someone carrying a light pulse clock moves by us at a speed v , we will see that their light pulse clock runs slow by a factor $\sqrt{1 - v^2/c^2}$. Up until now we have said nothing about any other kind of clock, and we have the problem that no one has actually constructed a light pulse clock. But we can easily generalize our result with another thought experiment fairly similar to the one we just did.

For the new thought experiment let us rejoin the discussion between the astronaut and the class of students. We begin just after the students have told the astronaut what they saw. "I was afraid of that," the astronaut replies. "I never did trust that light pulse clock. I am not at all surprised that it ran slow. But now my digital watch, it's really good. It is based on a quartz crystal and keeps really good time. It wouldn't run slow like the light pulse clock."

“I’ll bet it would,” Bill interrupts. “How much?,” the astronaut responds indignantly. “The cost of one more trip,” Bill answers.

In the new trip, the astronaut is to place his digital watch right next to the light pulse clock so that the astronaut and the class can see both the digital watch and the light pulse clock at the same time. The idea is to compare the rates of the two timing devices.

“Look what would happen,” Bill continues, “if your digital watch did not slow down. When you come by, your digital watch would be keeping “God’s time” as you call it, while your light pulse clock would be running slower.”

“The important part of this experiment is that because the faces of the two clocks are together, if we see them running at different rates, you will too. *You would notice that here on earth, when you are at rest, the two clocks ran at the same rate. But when you were moving at high speed, they would run at different rates. You could use this difference in rates to detect your own motion, and therefore violate the principle of relativity.*”

The astronaut thought about this for a bit, and then responded, “I’ll grant that you are partly right. On my previous trips, after the accelerations ceased and I started coasting toward the class, I did not feel any motion. I had no trouble unpacking the equipment and setting it up. The light pulse went in just as it had back in the lab, and I was sure that the light pulse clock was working just fine. I certainly would have noticed any difference in the rates of the two clocks.”

“Are you insinuating,” the astronaut continued, “that the reason I did not detect my light pulse running slow was because my digital watch was also running slow?”

“Almost,” replied Bill, “but you have other timing devices in your capsule. You shave once a day because you do not like the feel of a beard. This is a cyclic process that could be used as the basis of a new kind of clock. If your shaving cycle clock did not slow down just like the light pulse clock, you could time your shaving cycle with the light pulse clock and detect your motion. You would notice that you had to shave more times per light pulse month when you were moving than when you were at rest. This would violate the principle of relativity.”

“**Wow,**” the astronaut exclaimed, “*if the principle of relativity is correct, and the light pulse clock runs slow, then every process, all timing devices in my ship have to run slow in precisely the same way so that I cannot detect the motion of the ship.*”

The astronaut’s observation highlights the power and generality of the principle of relativity. It turns a limited theory about the behavior of one special kind of clock into a general theory about the behavior of all possible clocks. If the light pulse clock in the astronaut’s capsule is running slow by a factor $\sqrt{1 - v^2/c^2}$, then all clocks must run slow by exactly the same factor so that the astronaut cannot detect his motion.

Real Clocks

Our theory still has a severe limitation. We have to assume that the light pulse clock runs slow. But no one has yet built a light pulse clock. Thus our theory is still based on thought experiments and conjectures about the behavior of light. If we had just one real clock that ran slow by a factor $\sqrt{1 - v^2/c^2}$, then the principle of relativity would guarantee that all other clocks ran slow in precisely the same way. Then we would not need any conjectures about the behavior of light. The principle of relativity would do it all!

In 1905 when Einstein proposed the special theory of relativity, he did not have any examples of moving clocks that were observed to run slow. He had to rely on his intuition and the two postulates. It was not until the early 1930s, in studies of the behavior of an elementary particle called the *muon*, that experimental evidence was obtained showing that a real moving clock actually ran slow.

A muon at rest has a half life of 2.2 microseconds or 2,200 nanoseconds. That means that if we start with 1000 muons, 2.2 microseconds later about half will have decayed and only about 500 will be left. Wait another 2.2 microseconds and half of the remaining muons will decay and we will have only about 250 left, etc. If we wait 5 half lives, just over 10 microseconds, only one out of 32 of the original particles remain ($1/2 \times 1/2 \times 1/2 \times 1/2 \times 1/2 = 1/32$).

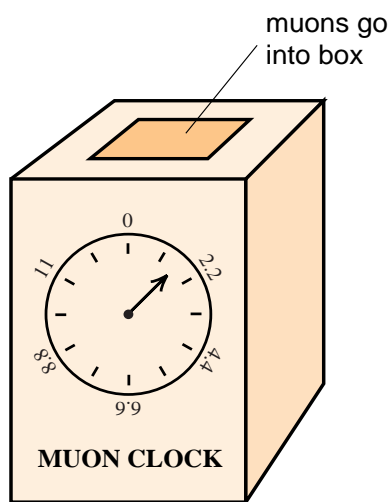


Figure 19

In our muon clock, every time half of the muons inside decay, we replace them and move the hand on the face forward by 2.2 microseconds.

Muons are created when cosmic rays from outer space strike the upper atmosphere. Few cosmic rays make it down to the lower atmosphere, so that most muons are created in the upper atmosphere, several miles up. The interesting results, observed in the 1930s was that there were almost as many high energy muons striking the surface of the earth as there were several miles up. This indicated that most of the high energy muons seemed to be surviving the several mile trip down through the earth's atmosphere.

Suppose we have a muon traveling at almost the speed of light, almost 1 foot per nanosecond. To go a mile, 5280 feet, would take 5,280 nanoseconds or about 5 microseconds. Therefore a 2 mile trip takes at least 10 microseconds, which is 5 half lives. One would expect that in this 2 mile trip, only one out of every 32 muons that started the trip would survive. Yet the evidence was that most of the high energy muons, those traveling close to the speed of light, survived. How did they do this?

We can get an idea of why the muons survive when we realize that the muon half life can be used as a timing device for a clock. Imagine that we have a box with a dial on the front as shown in Figure (19). We set the hand to 0 and put 1000 muons in the box. We wait until half the muons decay, whereupon we advance the hand 2.2 microseconds, replace the decayed muons so that we again have 1000 muons, and then wait until half have decayed again. If we keep repeating this process the hand will advance one muon half life in each cycle. Here we have a clock based on the muon half life rather than the swings of a pendulum or the vibrations of a quartz crystal.

The fact that most high energy muons raining down through the atmosphere survive the trip means that their half life is in excess of 10 microseconds, much longer than the 2.2 microsecond half life of a muon at rest. A clock based on these moving muons would run much slower than a muon clock at rest. Thus the experimental observation that the muons survive the trip down through the atmosphere gives us our first example of a real clock that runs slow when moving.

In the early 1960s, a motion picture was made that carefully studied the decay of muons in the trip down from the top of Mount Washington in New Hampshire to sea level (the sea level measurements were made in Cambridge, Massachusetts), a trip of about 6000 feet. Muons traveling at a speed of $v = .994c$ were studied and from the number surviving the trip, it was determined that the muon half life was lengthened to about 20 microseconds, a factor of 9 times longer than the 2.2 microsecond lifetime of muons at rest. Since $1/\sqrt{1 - v^2/c^2} = 9$ for $v = .994c$, a result we got back in Equation (16), we see that the moving picture provides an explicit example of a moving clock that runs slow by a factor $\sqrt{1 - v^2/c^2}$.

At the present time there are two ways to observe the slowing down of real clocks. One is to use elementary particles like the muon, whose lifetimes are lengthened significantly when the particle moves at nearly the speed of light. The second way is to use modern atomic clocks which are so accurate that one can detect the tiny slowing down that occurs when the clock rides on a commercial jet. We calculated that a clock traveling

500 miles per hour should lose one nanosecond every hour. This loss was detected to an accuracy of 1% when physicists at the University of Maryland in the early 1980s flew an atomic clock for 15 hours over Chesapeake Bay.

In more recent times atomic clocks have become so accurate that the slowing down of the clock has become a nuisance. When these clocks are moved from one location to another, they have to be corrected for the time that was lost due to their motion. For these clocks, even a one nanosecond error is too much.

Thus today the slowing down of moving clocks is no longer a hypothesis but a common observational fact. The slowing down by $\sqrt{1 - v^2/c^2}$ has been seen both for clocks moving at the slow speeds of a commercial jet and the high speeds travelled by elementary particles. We now have real clocks that run slow by a factor $\sqrt{1 - v^2/c^2}$ and no longer need to hypothesize about the behavior of light pulses. *All of our conjectures in this chapter hinge on the principle of relativity alone.*



Figure 19a -- Muon Lifetime Movie

The lifetimes of 568 muons, traveling at a speed of .994c, were plotted as vertical lines. If the muon's clocks did not run slow, these lines would show how far the muons could travel before decaying. One can see that very few of the muons would survive the trip from the top of Mt. Washington to sea level. Yet the majority do survive.

Movie

To play the movie, click the cursor in the photo to the left. Use up or down arrows on the keyboard to raise or lower volume. Left and right arrows step one frame forward or back and esc stops it. The movie is 36 minutes long.

Time Dilation

If all moving clocks run slow, does *time itself* run slow for the moving observer? That raises the question of how we define time. If time is nothing more than what we measure by clocks, and all clocks run slow, we might as well say that time runs slow. And we can give this effect a name like time dilation, the word dilation referring to the stretching out of seconds in a moving clock.

But time is such a personal concept, it plays such a basic role in our lives, that it seems almost demeaning that time should be nothing more than what we measure by clocks. We have all had the experience that time runs slow when we are bored, and fast when we are busy. Time is associated with all aspects of our life, including death. Can such an important concept be abstracted to be nothing more than the results of a series of measurements?

Let us take the following point of view. Let physicists' time be that which is measured by clocks. Physicists' time is what runs slow for an object moving by. If your sense of time does not agree with physicists' time, think of that as a challenge. Try to devise some experiment to show that your sense of time is measurably different from physicists' time. If it is, you might be able to devise an experiment that violates the principle of relativity.

Space Travel

In human terms, time dilation should have its greatest effect on space travelers who need to travel long distances and therefore must go at high speeds. To get an idea of the distances involved in space travel, we note that light takes 1.25 seconds to travel from the earth to the moon (the moon is 1.25 billion feet away), and 8 minutes to travel from the sun to the earth. We can say that the moon is 1.25 light seconds away and the sun is 8 light minutes distant.

Currently Neptune is the most distant planet (Pluto will be the most distant again in a few years). When Voyager II passed Neptune, the television signals from Voyager, which travel at the speed of light, took 2.5 hours to reach us. Thus our solar system has a radius of 2.5 light hours. It takes 4 years for light to reach us from the nearest star from our sun; stars are typically one to a few light years apart.

If you look up at the sky at night and can see the Milky Way, you will see part of our galaxy, a spiral structure of stars that looks much like our neighboring galaxy Andromeda shown in Figure (20). Galaxies are about 100,000 light years across, and typically spaced about a

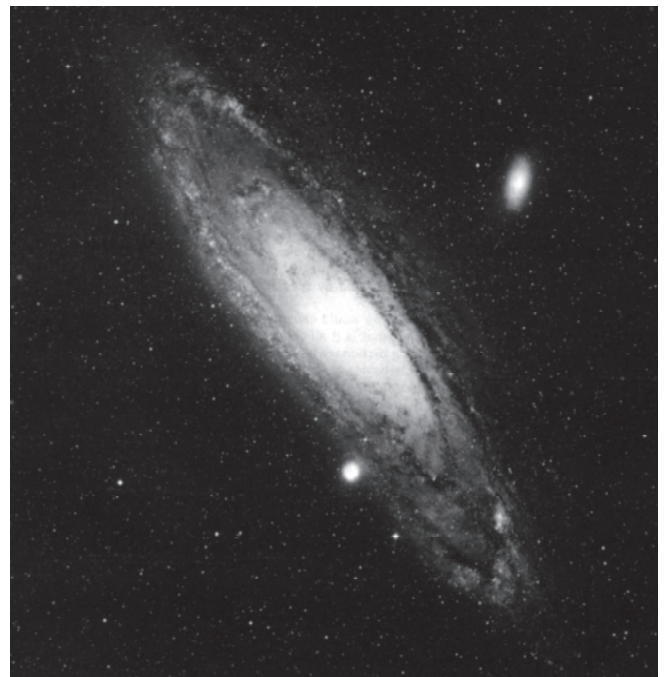


Figure 20
The Andromeda galaxy, about a million light years away, and about 1/10 million light years in diameter.

million light years apart. As we will see there are even larger structures in space; there are interesting things to study on an even grander scale.

Could anyone who is reading this text survive a trip to explore our neighboring galaxy Andromeda, or just survive a trip to some neighboring star, say, only 200 light years away?

Before Einstein's theory, one would guess that the best way to get to a distant star would be to go so fast that the trip would not take very long. But now we have a problem. In Einstein's theory, the speed of light is a special speed. If we had the astronaut carry our light pulse clock at a speed greater than the speed of light, the light pulse could not remain in the clock. The astronaut would also notice that he could not keep the light pulse in the clock, and could use that fact to detect his own uniform motion. In other words, the principle of relativity implies that we or the astronaut cannot travel faster than the speed of light.

That the speed of light is a limiting speed is common knowledge to physicists working with elementary particles. Small particle accelerators about a meter in diameter can accelerate electrons up to speeds approaching $v = .9999c$. The two mile-long accelerator at Stanford University, which holds the speed record for accelerating elementary particles here on earth, can only get electrons up to a speed $v = .999999999c$. The speed of light is Nature's speed limit, how this speed limit is enforced is discussed in Chapter 6.

Does Einstein's theory preclude the possibility that we could visit a distant world in our lifetime; are we confined to our local neighborhood of stars by Nature's speed limit? The behavior of the muons raining down through the atmosphere suggests that we are not confined. The muons, you will recall, live only 2.2 microseconds (on the average) when at rest. Yet the muons go much farther than the 2200 feet that light could travel in a muon half life. They survive the trip down through the atmosphere because their clocks are running slow.

If humans could accompany muons on a trip at a speed $v = .994c$, the human clocks should also run slow, their lifetimes should also expand by the same factor of 9. If the human clocks did not run slow and the muon clocks did, the difference in rates could be used to detect uniform motion in violation of the principle of relativity.

The survival of the muons suggest that we should be able to travel to a distant star in our own lifetime. Suppose, for example, we wish to travel to the star Zeta (we made up that name) which is 200 light years away. If we traveled at the speed $v = .994c$, our clocks should run slow by a factor $1/\sqrt{1 - v^2/c^2} = 1/9$, and the trip should only take us $200 \times 1/9 = 22.4$ years. We would be only 22.4 years older when we get there. A healthy, young crew should be able to survive that.

The Lorentz Contraction

A careful study of this proposed trip to star Zeta uncovers a consequence of Einstein's theory that we have not discussed so far. To see what this effect is, to see that it is just as real as the slowing down of moving clocks, we will treat this proposed trip as a new thought experiment which will be analyzed from several points of view.

In this thought experiment, the instructor and the class, who participated in the previous thought experiments, decide to travel to Zeta at a speed of $v = .994c$. They have a space ship constructed which on the inside looks just like their classroom, so that classroom discussions can be continued during the trip.

On the earth, a permanent government subagency of NASA is established to record transmissions from the space capsule and maintain an earth bound log of the trip. Since the capsule, traveling at less than the speed of light, will take over 200 years to get to Zeta, and since the transmissions upon arrival will take 200 years to get back, the NASA agency has to remain in operation for over 400 years to complete its assignment. NASA's summary of the trip, written in the year 2406, reads as follows: "The spacecraft took off in the year 2001 and spent four years accelerating up to a speed of $v = .994c$. During this acceleration everything was packed away, but when they got up to the desired speed, the rocket engines were shut off and they started the long coast to the Zeta. This coast started with a close fly-by of the earth in late January of the year 2005. The NASA mission control officer who recorded the fly-by noted that his great, great, great, grandchildren would be alive when the spacecraft reached its destination."

The mission control officer then wrote down the following calculations that were later verified in detail. "The spacecraft is traveling at a speed $v = .994c$, so that it will take 1/.994 times longer than it takes a pulse of light to reach the star. Since the star is 200 light years away, the spacecraft should take $200/.994 = 201.2$ years to get there.

But the passengers inside are also moving at a speed $v = .994c$, their clocks and biological processes run slow by a factor $\sqrt{1 - v^2/c^2} = 1/9$, and the amount of time they will age is

$$\begin{array}{l} \text{amount of time} \\ \text{space travelers} \\ \text{age} \end{array} = 201.2 \text{ years} \times \frac{1}{9} = 22.4 \text{ years}$$

Even the oldest member of the crew, the instructor, will be able to survive."

The 2406 entry continued; "During the intervening years we maintained communication with the capsule and everything seemed to go well. There were some complaints about our interpretation of what was happening but that did not matter, everything worked out just as we had predicted. The spacecraft flew past Zeta in March of the year 2206, and we received the communications of the arrival this past March. The instructor said she planned to retire after they decelerated and the spacecraft landed on a planet orbiting Zeta. She was not quite sure what her class of middle aged students would do."

NASA's predictions may have come true, but from the point of view of the class in the capsule, not everything worked out the way NASA said it did.

As NASA mentioned, a few years were spent accelerating the space capsule to the speed $v = .994c$. The orbit was chosen so that just after the engines were shut off and the coast to Zeta began, the spacecraft would pass close to the earth for one final good-by.

There was quite a change from the acceleration phase to the coasting phase. During the acceleration everything had to be securely fastened, and there was the constant vibration of the engines. But when the engines were shut off, you couldn't feel motion any more; everything floated as in the TV pictures of the early astronauts orbiting the earth.

When the coasting started, the instructor and class settled down to the business of monitoring the trip. The first step was to test the principle of relativity. Was there any experiment that they could do inside the capsule that could detect the motion of the capsule? Various experiments were tried, but none demonstrated that the capsule itself was moving. As a result the students voted to take the point of view that they and the capsule were at rest, and the things outside were moving by.

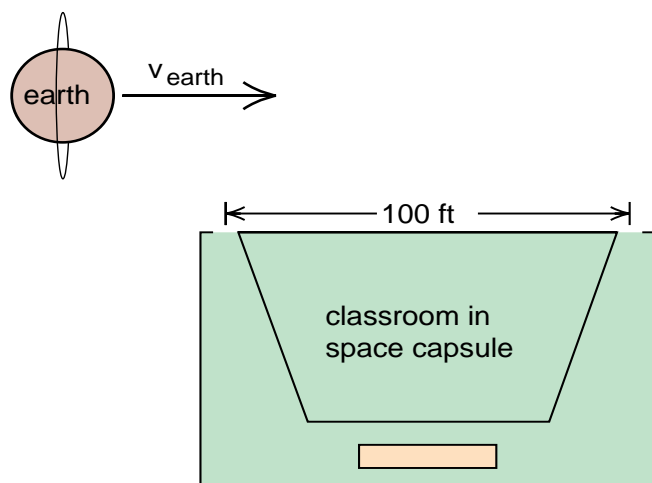


Figure 21

To measure the speed of the earth as it passes by, the class measures the time it takes a small satellite to pass by the windows in the back of the classroom. The windows are 100 feet apart.

Very shortly after the engines were shut off, the earth went by. This was expected, and the students were ready to measure the speed of the earth as it passed. There were two windows 100 feet apart on the back wall of the classroom, as shown in Figure (21). When the earth came by, there was an orbiting spacecraft, essentially at rest relative to the earth, that passed close to the windows. The students measured the time it took the front edge of this orbiting craft to travel the 100 feet between the windows. They got 100.6 nanoseconds and therefore concluded that the orbiting craft and the earth itself were moving by at a speed

$$\begin{aligned} v_{\text{earth}} &= \frac{100 \text{ feet}}{100.6 \text{ nanoseconds}} \\ &= .994 \frac{\text{feet}}{\text{nanosecond}} = .994 c \end{aligned}$$

So far so good. That was supposed to be the relative speed of the earth.

In the first communications with earth, NASA mission control said that the space capsule passed by the earth at noon, January 17, 2005. Since all the accurate clocks had been dismantled to protect them from the acceleration, and only put back together when the coasting started, the class was not positive about what time it was. They were willing to accept NASA's statement that the fly-by occurred on January 17, 2005. From then on, however, the class had their own clocks in order—light pulse clocks, digital clocks and an atomic clock. From then on they would keep their own time.

For the next 22 years the trip went smoothly. There were numerous activities, video movies, etc., to keep the class occupied. Occasionally, about once every other month, a star went by. As each star passed, its speed v was measured and the class always got the answer $v = .994c$. This confirmed that the earth and the neighboring stars were all moving together like bright dots on a huge moving wall.

The big day was June 13, 2027, the 45th birthday of Jill who was eighteen when the trip was planned. This was the day, 22.4 years after the earth fly-by, that Zeta went by. The students made one more speed measurement and determined that Zeta went by at a speed $v = .994c$. An arrival message was sent to NASA, one day was allowed for summary discussions of the trip, and then the deceleration was begun.

After a toast to Jill for her birthday, Bill began the conversation. “Over the past few years, the NASA communications and even our original plans for the trip have been bothering me. The star charts say that Zeta is 200 light years from the earth, but that cannot be true.”

“Look at the problem this way.” Bill continues. “The earth went by us at noon on January 17, 2005, just 22.4 years ago. When the earth went by, we observed that it took 100.6 nanoseconds to pass by our 100 foot wide classroom. Thus the earth went by at a speed $v = .994$ feet/nanosecond, or $.994c$. Where is the earth now, 22.4 years later? How far could the earth have gotten, traveling at a speed $.994c$ for 22.4 years? My answer is

$$\begin{aligned} \text{distance of earth} &= .994 \frac{\text{light year}}{\text{year}} \times 22.4 \text{ years} \\ \text{from spaceship} &= 22 \text{ light years} \end{aligned}$$

“You’re right!” Joan interrupted, “Even if the earth had gone by at the speed of light, it would have gone only 22.4 light years in the 22.4 years since fly-by. The star chart must be wrong.”

The instructor, who had just entered the room, said, “I object to that remark. As a graduate student I sat in on part of a course in astronomy and they described how the distance to Zeta was measured.” The instructor drew a sketch, Figure (22), and continued. “Here is the earth in its orbit about the sun, and two observations, six months apart, are made of Zeta. You see that the two positions of the earth and the star form a triangle. Telescopes can accurately measure the two angles I labeled θ_1 and θ_2 , and the distance across the earth’s orbit is accurately known to be 16 light minutes. If you know two angles and one side of a triangle, then you can calculate the other sides from simple geometry. One reason for choosing a trip to Zeta is that we had accurate

measurements of the distance to that star. We knew that it was 200 light years away, and we knew that traveling at a speed $.994c$, we could survive the trip in our lifetime.”

Bill responded, “I think you entered the room too late and missed my argument. Let me summarize it. *Point 1:* the earth went by a little over 22 years ago. *Point 2:* we actually measured that the earth was traveling by us at almost the speed of light. *Point 3:* even light cannot go farther than 22 light years in 22 years. The earth can be no farther than about 22 light years away. *Point 4:* Zeta passed by us today, thus *the distance from the earth to Zeta is about 22 light years, not 200 light years!*”

“But what about NASA’s calculations and all their plans,” the instructor said, interrupting a bit nervously.

“We do not care what NASA thinks,” responded Bill. “We have had no acceleration since the earth went by. Thus the principle of relativity guarantees that we can take the point of view that we are at rest and that it is the earth and NASA that are moving. From our point of view, the earth is 22 light years away. What NASA thinks is their business.”

Joan interrupts, “Let us not argue on this last day. Let’s figure out what is happening. There is something more important here than just how far away the earth is.”

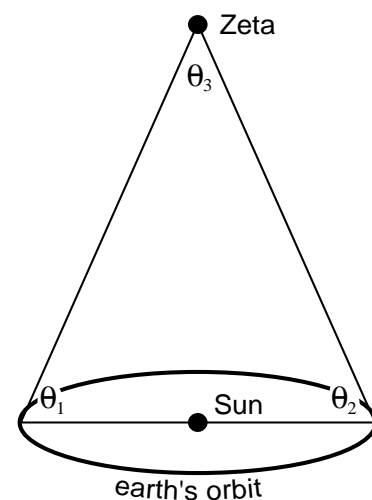


Figure 22
Instructor's sketch showing how the distance from the earth to the star Zeta was measured. (For a star 200 light years away, θ_3 is 4.5 millionths of a degree.)

“Remember in the old lectures on time dilation where the astronaut carried a light pulse clock. We used the peculiar behavior of that clock and the principle of relativity to deduce that time ran slow for a moving observer.”

“Now for us, NASA is the moving observer. More than that, the earth, sun, and the stars, including Zeta, have all passed us going in the same direction and the same speed $v = .994c$. We can think of them as all in the same huge space ship. Or we can think of the earth and the stars as painted dots on a very long rod. A very long rod moving past us at a speed $v = .994c$. See my sketch (Figure 23).”

“To NASA, and the people on earth, this huge rod, with the sun at one end and Zeta at the other, is 200 light years long. Our instructor showed us how earth people measured the length of the rod. But as Bill has pointed out, to us this huge rod is only 22 light years long. That moving rod is only 1/9th as long as the earth people think it is.”

“But,” Bill interrupts, the factor of 1/9 is exactly the factor $\sqrt{1 - v^2/c^2}$ by which the earth people thought our clocks were running slow. Everyone sees something peculiar. The earth people see our clocks running slow by a factor $\sqrt{1 - v^2/c^2}$, and we see this hypothetical rod stretched from the sun to Zeta contracted by a factor $\sqrt{1 - v^2/c^2}$.”

“But I still worry about the peculiar rod of Joan’s,” Bill continues, “what about real rods, meter sticks, and so forth? Will they also contract?”

At this point Joan sees the answer to that. “Remember, Bill, when we first discussed moving clocks, we had only the very peculiar light pulse clock that ran slow. But then we could argue that all clocks, no matter how they are constructed, had to run slow in exactly the same way, or we could violate the principle of relativity.”

“We have just seen that my ‘peculiar’ rod, as you call it, contracts by a factor $\sqrt{1 - v^2/c^2}$. We should be able to show with some thought experiments that all rods, no matter what they are made of, must contract in exactly the same way as my peculiar one or we could violate the principle of relativity.”

“That’s easy,” replies Bill. Just imagine that we string high tensile carbon filament meter sticks between the sun and Zeta. I estimate (after a short calculation) that it should take only 6×10^{17} of them. As we go on our trip, it doesn’t make any difference whether the meter sticks are there or not, everything between the earth and Zeta passes by in 22 years. We still see 6×10^{17} meter sticks. But each one must have shortened by a factor $\sqrt{1 - v^2/c^2}$ so that all of them fit in the shortened distance of 22 light years. It does not make any difference what the sticks are made of.”

Jim, who had not said much up until now, said, “OK, from your arguments I see that the length of the meter sticks, the length in the direction of motion must contract by a factor $\sqrt{1 - v^2/c^2}$, but what about the width? Do the meter sticks get skinnier too?”

The class decided that Jim’s question was an excellent one, and that a new thought experiment was needed to decide.

Let’s try this,” suggested Joan. “Imagine that we have a space ship 10 feet in diameter and we build a brick wall with a circular hole in it 10 feet in diameter (Figure (24)). Let us assume that widths, as well as lengths, contract. To test the hypothesis, we hire an astronaut to

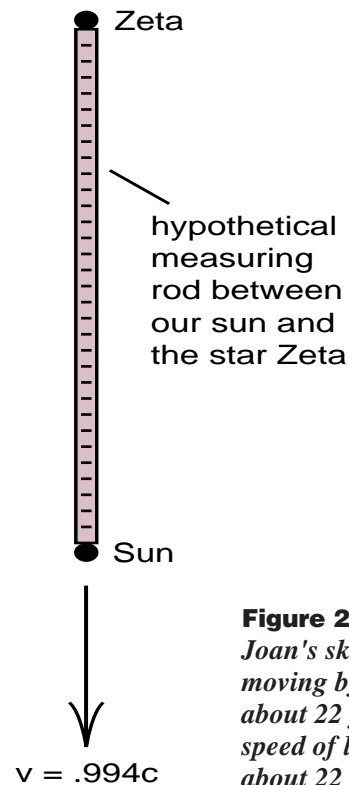


Figure 23

Joan's sketch of the Sun and Zeta moving by. This “object” passed by in about 22 years, moving at nearly the speed of light. Thus the “object” was about 22 light years long.

fly the 10 foot diameter capsule through the 10 foot hole at nearly the speed of light, say at $v = .994c$. If widths contract like lengths, the capsule should contract to 10/9 of a foot; it should be just over 13 inches in diameter when it gets to the 10 foot hole. It should have no trouble getting through.”

“But look at the situation from the astronaut’s point of view. He is sitting there at rest, and a brick wall is approaching him at a speed $v = .994c$. He has been told that there is a 10 foot hole in the wall, but he has also been told that the width of things contracts by a factor $\sqrt{1 - v^2/c^2}$. That means that the diameter of the hole should contract from 10 feet to 13 inches. He is sitting there in a 10 foot diameter capsule, a brick wall with a 13 inch hole is approaching him, and he is supposed to fit through. No way! He bails out and looks for another job.”

“That’s a good way to do thought experiments, Joan,” replied the instructor. “Assume that what you want to test is correct, and then see if you can come up with an inconsistency. In this case, by assuming that widths contract, you predicted that the astronaut should easily make it through the hole in the wall. But the astronaut faced disaster. The crash, from the astronaut’s point of view would have been an unfortunate violation of the principle of relativity, which he could use as evidence of his own uniform motion.”

“To sum it up,” the instructor added, “we now have time dilation where moving clocks run slow by a factor $\sqrt{1 - v^2/c^2}$, and we see that moving lengths contract by the same factor. Only lengths in the direction of motion contract, widths are unchanged.”

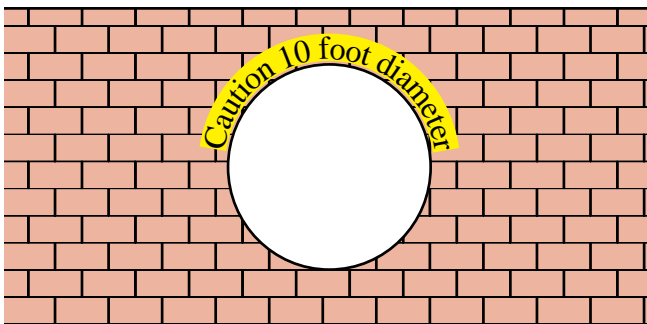


Figure 24
Do diameters contract?

Leaving our thought experiment, it is interesting to note that the discovery of the contraction of moving lengths occurred before Einstein put forth the special theory of relativity. In the 1890s, physicist George Fitzgerald assumed that the length of one of the arms in Michaelson’s interferometer, the arm along the direction of motion, contracted by a factor $\sqrt{1 - v^2/c^2}$. This was just the factor needed to keep the interferometer from detecting the earth’s motion in the Michaelson-Morley experiments. It was a short while later that H.A. Lorentz showed that if the atoms in the arm of the Michaelson interferometer were held together by electric forces, then such a contraction would follow from Maxwell’s theory of electricity. The big step, however, was Einstein’s assumption that the principle of relativity is correct. Then, if one object happens to contract when moving, all objects must contract in exactly the same way so that the contraction could not be used to detect one’s own motion. This contraction is called the Lorentz-Fitzgerald contraction, or Lorentz contraction, for short.

Relativistic Calculations

Although we have not quite finished with our discussion of Einstein’s special theory of relativity, we have covered two of the important consequences, time dilation and the Lorentz contraction, which will play important roles throughout the text. At this point we will take a short break to discuss easy ways to handle calculations involving these relativistic effects. Then we will take another look at Einstein’s theory to see if there are any more new effects to be discovered.

After our discussion of time dilation, we pointed out the importance of the quantity $\sqrt{1 - v^2/c^2}$ which is a number always less than 1. If we wanted to know how much longer a moving observer’s time interval was, we divided by $\sqrt{1 - v^2/c^2}$ to get a bigger number. If we wanted to know how much less was the frequency of a moving clock, we multiplied by $\sqrt{1 - v^2/c^2}$ to get a smaller number.

With the Lorentz contraction we have another effect that depends upon $\sqrt{1 - v^2/c^2}$. If we see an object go by us, the object will contract in length. To predict its contracted length, we multiply the uncontracted length by $\sqrt{1 - v^2/c^2}$ to get a smaller number. If, on the other

hand, an object moving by us had a contracted length l , and we stop the object, the contraction is undone and the length increases. We get the bigger uncontracted length by dividing by $\sqrt{1 - v^2/c^2}$.

As we mentioned earlier, first determine intuitively whether the number gets bigger or smaller, then either multiply by or divide by the $\sqrt{1 - v^2/c^2}$ as appropriate. This always works for time dilation, the Lorentz contraction, and, as we shall see later, relativistic mass.

We will now work some examples involving the Lorentz contraction to become familiar with how to handle this effect.

Example 1 Muons and Mt Washington

In the Mt. Washington experiment, muons travel 6000 feet from the top of Mt. Washington to sea level at a speed $v = .994c$. Most of the muons survive despite the fact that the trip should take about 6 microseconds (6000 nanoseconds), and the muon half life is $\tau = 2.2$ microseconds for muons at rest.

We say that the muons survive the trip because their internal timing device runs slow and their half life expands by a factor $1/\sqrt{1 - v^2/c^2} = 9$. The half life of the moving muons should be

$$\begin{aligned} \text{half life of} &= \frac{\tau}{\sqrt{1 - v^2/c^2}} \\ \text{moving muons} &= 2.2 \text{ microseconds} \times 9 \\ &= 19.8 \text{ microseconds} \end{aligned}$$

This is plenty of time for the muons to make the trip.

From the muon's point of view, they are sitting at rest and it is Mt. Washington that is going by at a speed $v = .994c$. The muon's clocks aren't running slow, instead the height of Mt. Washington is contracted.

To calculate the contracted length of the mountain, start with the 6000 foot uncontracted length, multiply by $\sqrt{1 - v^2/c^2} = 1/9$ to get

$$\begin{aligned} \text{contracted height} &= 6000 \text{ feet} \times \frac{1}{9} \\ \text{of Mt. Washington} &= 667 \text{ feet} \end{aligned}$$

Traveling by at nearly the speed of light, the 667 foot high Mt. Washington should take about 667 nanoseconds or .667 microseconds to go by. Since this is considerably less than the 2.2 microsecond half life of the muons, most of them should survive until sea level comes by.

Example 2 Slow Speeds

Joan walks by us slowly, carrying a meter stick pointing in the direction of her motion. If her speed is $v = 1$ foot/second, what is the contracted length of her meter stick as we see it?

This is an easy problem to set up. Since her meter stick is contracted, we multiply 1 meter times the $\sqrt{1 - v^2/c^2}$ with $v = 1$ foot/second. The problem comes in evaluating the numbers. Noting that 1 nanosecond = 10^{-9} seconds, we can use the conversion factor 10^{-9} seconds/nanosecond to write

$$\begin{aligned} v &= 1 \frac{\text{ft}}{\text{sec}} \times 10^{-9} \frac{\text{sec}}{\text{nanosecond}} \\ &= 10^{-9} \frac{\text{ft}}{\text{nanosecond}} = 10^{-9} c \end{aligned}$$

Thus we have

$$\frac{v}{c} = 10^{-9}, \quad \frac{v^2}{c^2} = 10^{-18}$$

and for Joan's slow walk we have

$$\sqrt{1 - v^2/c^2} = \sqrt{1 - 10^{-18}} \quad (19)$$

If we try to use a calculator to evaluate the square root in Equation (19), we get the answer 1. For the calculator, the number 10^{-18} is so small compared to 1, that it is ignored. It is as if the calculator is telling us that when Joan's meter stick is moving by at only 1 foot/second, there is no noticeable contraction.

But there is some contraction, and we may want to know the contraction no matter how small it is. Since calculators cannot handle numbers like $1 - 10^{-18}$, we need some other way to deal with such expressions. For this, there is a convenient set of approximation formulas which we will now derive.

Approximation Formulas

The approximation formulas deal with numbers close to 1, numbers that can be written in the form $(1 + a)$ or $(1 - a)$ where a is a number much less than 1. For example the square root in Equation (19) can be written as

$$\sqrt{1 - 10^{-18}} = \sqrt{1 - \alpha}$$

where $\alpha = 10^{-18}$ is truly a number much less than 1.

The idea behind the approximation formulas is that if a is much less than 1, a^2 is very much less than 1 and can be neglected. To see how this works, let us calculate $(1 + a)^2$ and see how we can neglect a^2 terms even when a is as large as .01. An exact calculation is

$$(1 + \alpha)^2 = 1 + 2\alpha + \alpha^2$$

which for $\alpha = .01$, $\alpha^2 = .0001$ is

$$\begin{aligned} (1 + \alpha)^2 &= 1 + .02 + .0001 \\ &= 1.0201 \end{aligned}$$

If we want to know how much $(1 + \alpha)^2$ differs from 1, but do not need too much precision, we could round off 1.0201 to 1.02 to get

$$(1 + \alpha)^2 \approx 1.02$$

(The symbol \approx means “approximately equal to”). But in replacing 1.0201 by 1.02, we are simply dropping the α^2 term in Equation (19). We can write

$$(1 + \alpha)^2 \approx 1 + 2\alpha = 1 + .02 = 1.02 \quad (20)$$

Equation (20) is our first example of an approximation formula.

In Equation (20) the smaller a is the better the approximation. If $a = .0001$ we have

$$(1.0001)^2 = 1.00020001 \quad (\text{exact})$$

Equation (20) gives

$$(1 + .0001)^2 \approx 1 + .0002 = 1.0002$$

and we see that the neglected α^2 terms become less and less important.

Some useful approximation formulas are the following

$$(1 + \alpha)^2 \approx 1 + 2\alpha \quad (20)$$

$$(1 - \alpha)^2 \approx 1 - 2\alpha \quad (21)$$

$$\frac{1}{1 + \alpha} \approx 1 - \alpha \quad (22)$$

$$\frac{1}{1 - \alpha} \approx 1 + \alpha \quad (23)$$

$$\sqrt{1 - \alpha} \approx 1 - \frac{\alpha}{2} \quad (24)$$

$$\frac{1}{\sqrt{1 - \alpha}} \approx 1 + \frac{\alpha}{2} \quad (25)$$

We have already derived Equation (20). Equation (21) follows from (20) if we replace α by $-\alpha$.

Equation (22) can be derived as follows. Multiply the quantity $1 - a$ by $(1+a)/(1+a)$ which is 1 to get

$$1 - \alpha = (1 - \alpha) \times \frac{(1 + \alpha)}{(1 + \alpha)} = \frac{(1 - \alpha^2)}{1 + \alpha} \approx \frac{1}{1 + \alpha}$$

In the last step we dropped the α^2 terms.

To derive the approximate formula for a square root, start with

$$\left(1 - \frac{\alpha}{2}\right) \times \left(1 - \frac{\alpha}{2}\right) = 1 - 2 \frac{\alpha}{2} + \frac{\alpha^2}{4} \approx 1 - \alpha \quad (26)$$

taking the square root of Equation (26) gives

$$1 - \frac{\alpha}{2} \approx \sqrt{1 - \alpha}$$

which is the desired result. Again we only neglected α^2 terms.

To derive Equation (25), first use Equation (24) to get

$$\frac{1}{\sqrt{1 - \alpha}} \approx \frac{1}{1 - \frac{\alpha}{2}}$$

Then use Equation (23), with a replaced by $a/2$ to get

$$\frac{1}{1 - \frac{\alpha}{2}} \approx 1 + \frac{\alpha}{2}$$

which is the desired result.

For those who are interested, the approximation formulas we have written are the first term of the so called *binomial expansion*:

$$(1 + \alpha)^n = 1 + n\alpha + \frac{n(n-1)}{2}\alpha^2 + \dots \quad (27)$$

where the coefficients of α , α^2 , etc. are known as the *binomial coefficients*. If you need more accurate approximations, you can use Equation (27) and keep terms in α^2 , α^3 , etc. For all the work in this text, the first term is adequate.

Exercise 5

Show that Equations (20) through (25) are all examples of the first order binomial expansion

$$(1 + \alpha)^n \approx 1 + n\alpha \quad (27a)$$

We are now ready to apply our approximation formulas to evaluate $\sqrt{1 - 10^{-18}}$ that appeared in Equation (17). Since $\alpha = 10^{-18}$ is very small compared to 1, we have

$$\sqrt{1 - 10^{-18}} = \sqrt{1 - \alpha} \approx 1 - \frac{\alpha}{2} = 1 - \frac{10^{-18}}{2}$$

Thus the length of Joan's meter stick is

$$\text{length of Joan's contracted meter stick} = 1 \text{ meter} \times \sqrt{1 - v^2/c^2}$$

$$= 1 \text{ meter} \left(1 - \frac{10^{-18}}{2} \right)$$

$$= 1 \text{ meter} - 5 \times 10^{-19} \text{ meters}$$

Exercise 6

We saw that time dilation in a commercial jet was not a big effect either—clocks losing only one nanosecond per hour in a jet traveling at 500 miles per hour. This was not an unnoticed effect, however, because modern atomic clocks can detect this loss.

In our derivation of the one nanosecond loss, we stated in Equation (17) that

$$\frac{1}{\sqrt{1 - v^2/c^2}} \approx 1 + 2.7 \times 10^{-13} \quad \text{for a speed of } 500 \text{ miles/hour} \quad (17)$$

Starting with

$$v = 500 \frac{\text{miles}}{\text{hour}} \times 5280 \frac{\text{feet}}{\text{mile}} \times \frac{1}{3600 \text{ sec/hour}}$$

use the approximation formulas to derive the result stated in Equation (17).

Exercise 7

Here is an exercise where you do not need the approximation formulas, but which should get you thinking about the Lorentz contraction. Suppose you observe that the Mars-17 spacecraft, traveling by you at a speed of $v = .995c$, passes you in 20 nanoseconds. Back on earth, the Mars-17 spacecraft is stored horizontally in a hanger that is the same length as the spacecraft. How long is the hanger?

A CONSISTENT THEORY

As we gain experience with Einstein's special theory of relativity, we begin to see a consistent pattern emerge. We are beginning to see that there is general agreement on what happens, even if different observers have different opinions as to *how* it happens. A good example is the Mt. Washington experiment observing muons traveling from the top of Mt. Washington to sea level. Everyone agrees that the muons made it. The muons are actually seen down at sea level. How they made it is where we get the differing points of view. We say that they made it because their clocks ran slow. They say they made it because the mountain was short. Time dilation is used from one point of view, the Lorentz contraction from another.

Do we have a complete, consistent theory now? In any new situation will we always agree on the predicted outcome of an experiment, even if the explanations of the outcome differ? Or are there some new effects, in addition to time dilation and the Lorentz contraction, that we will have to take into account?

The answer is that there is one more effect, called the *lack of simultaneity* which is a consequence of Einstein's theory. When we take into account this lack of simultaneity as well as time dilation and the Lorentz contraction, we get a completely consistent theory. Everyone will agree on the predicted outcome of any experiment involving uniform motion. No other new effects are needed to explain inconsistencies.

The lack of simultaneity turns out to be the biggest effect of special relativity, it involves two factors of $\sqrt{1 - v^2/c^2}$. But in this case the formulas are not as important as becoming familiar with some of the striking consequences. We will find ourselves dealing with problems such as whether we can get answers to questions that have not yet been asked, or whether gravity can crush matter out of existence. Strangely enough, these problems are related.

LACK OF SIMULTANEITY

One of the foundations of our intuitive sense of time is the concept of simultaneity. "Where were you when the murder was committed," the prosecutor asks. "At the time of the murder," the defendant replies, "I was eating dinner across town at Harvey's Restaurant." If the defendant can prove that the murder and eating dinner at Harvey's were simultaneous events, the jury will set him free. Everyone knows what simultaneous events are, or do they?

One of the most unsettling consequences of Einstein's theory is that the simultaneity of two events depends upon the point of view of the observer. Two events that from our point of view occurred simultaneously, may not be simultaneous to an observer moving by. Worse yet, two events that occurred one after the other to us, may have occurred in the reverse order to a moving observer.

To see what happens to the concept of simultaneous events, we will return to our thought experiment involving the instructor and the class. The action takes place on the earth before the trip to the star Zeta, and Joan has just brought in a paperback book on relativity.

"I couldn't understand that book either," the instructor says to Joan, "he starts with Einstein's analogy of trains and lightning bolts, but then switches to wind and sound waves, which completely confused me. There are many popular attempts to explain Einstein's theory, but most do not do very well when it comes to the lack of simultaneity."

"One of the problems with these popular accounts," the instructor continues, "is that we have to imagine too much. In today's lecture I will try to avoid that. In class we are going to carry out a real experiment involving two simultaneous events. We are going to discuss that experiment until everyone in class is completely clear about what happened. No imagining yet, just observe what actually occurred. When there are no questions left, then we will look at our real experiment from the point of view of someone moving by. At that point the main features of Einstein's theory are easy to see."

“The apparatus for our experiment is set up here on the lecture bench (Figure 25). On the left side of the bench I have a red flash bulb and on the right side a green flash bulb. These flash bulbs are attached to batteries and photocells so that when a light beam strikes their base, they go off.”

“In the center of the desk is a laser and in front of it a beam splitter that uses half silvered mirrors. When I turn the laser on, the laser beam comes out, strikes the beam splitter, and divides into two beams. One beam travels to the left and sets off the red flash bulb, while the other beam goes to the right and triggers the green flash bulb. I will call the beams emerging from the beam splitter ‘trigger beams’ or ‘trigger pulses’.”

“Let us analyze the experiment before we carry it out,” the instructor continues. “We will use the Einstein postulate that the speed of light is c to all observers. Thus the left trigger pulse travels at a speed c and so does the right one as I showed on the sketch. Since the beam splitter is in the center of the desk, the trigger pulses which start out together, travel the same distance at the same speeds to reach the flash bulbs. As a result the flash bulbs must go off simultaneously.”

“The flashing of the flashbulbs are an example of what I mean by *simultaneous events*,” the instructor adds with emphasis. “I know that they will be simultaneous events because of the way I set up the experiment”.

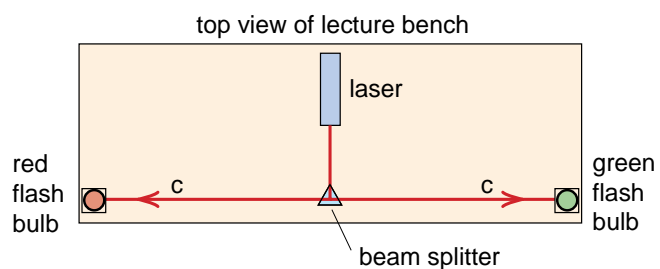


Figure 25

Lecture demonstration experiment in which two flashbulbs are fired simultaneously by trigger signals from a laser. The laser and beam splitter are at the center of the lecture bench, so that the laser light travels equal distances to reach the red and green bulbs. A photocell, battery and relay are mounted in each flashbulb base.

“OK, let’s do the experiment.”

While the instructor is adjusting the apparatus, one of the flashbulbs goes off accidentally which amuses the class, but finally the apparatus is ready, the laser beam turned on, and both bulbs fire.

“Well, were they simultaneous flashes?” the instructor asks the class.

“I guess so,” Bill responds, a bit hesitantly.

“How do you know,” the instructor asks.

“Because you set it up that way,” answers Bill.

Turning and pointing a finger at Joan who is sitting on the right side of the room nearer the green flash bulb (as in Figure 26), the instructor says, “Joan, for you which flash was first?” Joan thought for a second and replied, “The green bulb is closer, I should have seen the green light first.”

“But which occurred first?” the instructor interrupts.

“What are you trying to get at?” Joan asks.

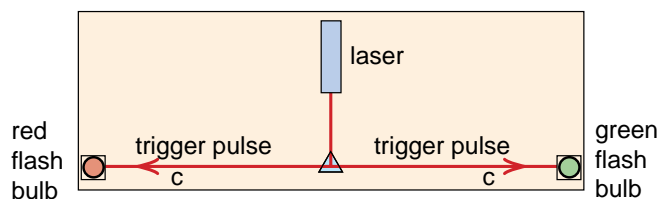


Figure 26

Although Joan sees the light from the green flash first, she knows that the two flashes were simultaneous because of the way the experiment was set up.



Joan

“Let me put it this way,” the instructor responds. “Around 1000 BC, the city of Troy fell to the invading Greek army. About the same time, a star at the center of the Crab Nebula exploded in what is known as a supernova explosion. Since the star is 2000 light years away, the light from the supernova explosion took 2000 years to get here. The light arrived on July 4, 1057, about the time of the Battle of Hastings. Now which are simultaneous events? The supernova explosion and the Battle of Hastings, or the supernova explosion and the fall of Troy?”

“I get the point,” replied Joan. “Just because they saw the light from the supernova explosion at the time of the Battle of Hastings, does not mean that the supernova explosion and that battle occurred at the same time. We have to calculate back and figure out that the supernova explosion occurred about the time the Greeks were attacking Troy, 2000 years before the light reached us.”

“As I sit here looking at your experiment,” Joan continues, “I see the light from the green flash before the light from the red flash, but I am closer to the green bulb than the red bulb. If I measure how much sooner the green light arrives, then measure the distances to the two bulbs, and do some calculations, I’ll probably find that the two flashes occurred at the same time.”

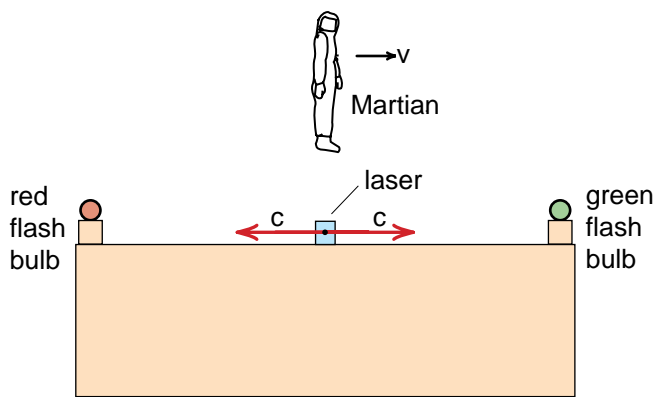


Figure 27a
In our thought experiment, a Martian astronaut passes by our lecture bench at a high speed v .

“It is much easier than that.” the instructor exclaimed, “Don’t worry about when the light reaches you, just look at the way I set up the experiment – two trigger pulses, starting at the same time, traveling the same distance at the same speed. The flashes must have occurred simultaneously. I chose this experiment because it is so easy to analyze when you look at the trigger pulses.”

“Any other questions?” the instructor asks. But by this time the class is ready to go on. “Now let us look at the experiment from the point of view of a Martian moving to the right a high speed v (Figure 27a). The Martian sees the lecture bench, laser, beam splitter and two flash bulbs all moving to the left as shown (Figure 27b). The lecture bench appears shortened by the Lorentz contraction, but the beam splitter is still in the middle of the bench. What is important is that the trigger pulses, being light, both travel outward from the beam splitter *at a speed c* .

As the bench passes by, the Martian sees that the green flash bulb quickly runs into the trigger pulse like this (). But on the other side there is a race between the trigger pulse and the red flash bulb, (), and the race continues for a long time after the green bulb has fired. For the Martian, the green bulb actually fired first, and the two flashes were not simultaneous.”

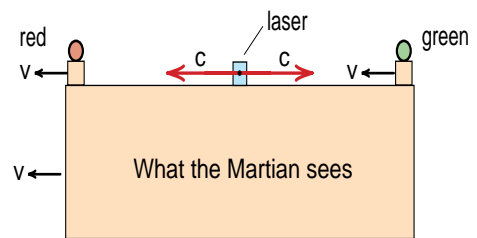


Figure 27b
The Martian astronaut sees the green flashbulb running into its trigger signal and firing quickly. The red flashbulb is running away from its trigger signal, and therefore will not fire for a long time. Clearly the green flash occurs first.

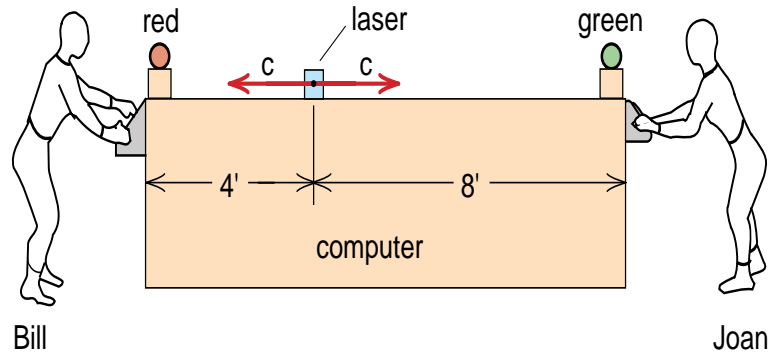


Figure 29
 To test the speed of the computer, Bill thinks of a question, and types it in, when he sees the red flash. Joan checks to see if the answer arrives at the same time as the green flash.

CAUSALITY

“You can reverse the order of two events that are years apart!” Bill exclaimed. “Couldn’t something weird happen in that time?”

“What about cause and effect,” asked Joan. “If you can reverse the order of events, can’t you reverse cause and effect? Can’t the effect come before the cause?”

“In physics,” the instructor responds, “there is a principle called *causality* which says that you cannot reverse cause and effect. Causality is not equivalent to the principle of relativity, but it is closely related, as we can see from the following thought experiment.”

“Suppose,” she said, “we read an ad for a brand new IBM computer that is really fast. The machine is so fast that when you type a question in at one end, the answer is printed out at the other end, 4 nanoseconds later. We look at the ad, see that the machine is 12 feet long, and order one to replace our lecture bench. After the machine is installed, we decide to test the accuracy of the ad. Do we really get answers in 4 nanoseconds? To find out, we set up the laser, beam splitter and flash bulbs on the computer instead of the lecture bench. The

main difference in the setup is that the laser and beam splitter have been moved from the center, over closer to the end where we type in questions. We have set it up so that the trigger pulse travels 4 feet to the red bulb and 8 feet to the green bulb as shown in the sketch (Figure 29). Since the trigger pulse takes 4 nanoseconds to get to the red bulb, and 8 nanoseconds to reach the green bulb, the red flash will go off 4 nanoseconds before the green one. We will use these 4 nanoseconds to time the speed of the computer.”

“Bill,” the instructor says, motioning to him, “you come over here, and when you see the red flash *think* of a question. Then type it into the machine. Do not think of the question until after you see the red flash, but then think of it and type it in quickly. We will assume that you can do that in much less than a nanosecond. You can always do that kind of thing in a thought experiment.”

“OK, Joan,” the instructor says, motioning to Joan, “you come over here and look for the answer to Bill’s question. If the ad is correct, if the machine is so fast that the answer comes out in 4 nanoseconds, then the answer should arrive when the green flash goes off.”

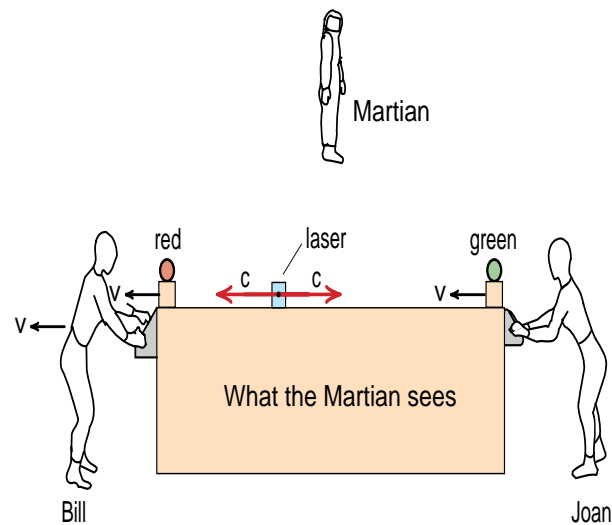


Figure 30
 To a Martian passing by, our computer is moving to the left at a speed near the speed of light. The race between the red bulb and its trigger signal takes so long that the green bulb fires first. As a result, Joan sees the answer to a question that Bill has not yet thought of. (This is what could happen if information travels faster than the speed of light.)

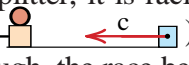
The instructor positions Bill and Joan and the equipment as shown in Figure (29), turns on the laser and fires the flash bulbs.

“Did you type in the question,” the instructor asks Bill, “when the red flash occurred?”

“Of course,” responds Bill, humoring the instructor.

“And did the answer arrive at the same time as the green flash,” the instructor asks Joan.

“Sure,” replies Joan, “why not?”

“Suppose it did,” replied the instructor. “Suppose the ad is right, and the answer is printed when the green flash goes off. Let us now look at this situation from the point of view of a Martian who is traveling to the right at a very high speed. The situation to the Martian looks like this (Figure 30). Although the red bulb is closer to the beam splitter, it is racing away from the trigger pulse ($v \leftarrow$ ). If the computer is going by fast enough, the race between the red bulb and its trigger pulse will take much longer than the head-on collision between the green bulb and its trigger pulse. The green flash will occur before the red flash.”

“And I,” interrupts Joan, “will see the answer to a question that Bill has not even thought of yet!”

“I thought you would be in real trouble if you could reverse the order of events,” Joan added.

“It is not really so bad,” the instructor continued. “If the ad is right, if the 12 foot long computer can produce answers that travel across the machine in 4 nanoseconds, we are in deep trouble. In that case we could see

answers to questions that have not yet been asked. That machine can be used to violate the principle of causality. But there was something peculiar about that machine. When the answer went through the machine, information went through the machine at three times the speed of light. Light takes 12 nanoseconds to cross the machine, while the answer went through in 4 nanoseconds.”

“Suppose,” asks Bill, “that the answer did not travel faster than light. Suppose it took 12 nanoseconds instead of 4 nanoseconds for the answer to come out.” The instructor replied, “To measure a 12 nanosecond delay with our flash bulb apparatus, we would have to set the beam splitter right up next to the red bulb like this (Figure 31) in order for the trigger signal to reach the green bulb 12 nanoseconds later. But with this setup, the red bulb flashes as soon as the laser is turned on. No one, no matter how they are moving by, sees a race between the red bulb and the trigger signal. Everybody agrees that the green flash occurs after the red flash.”

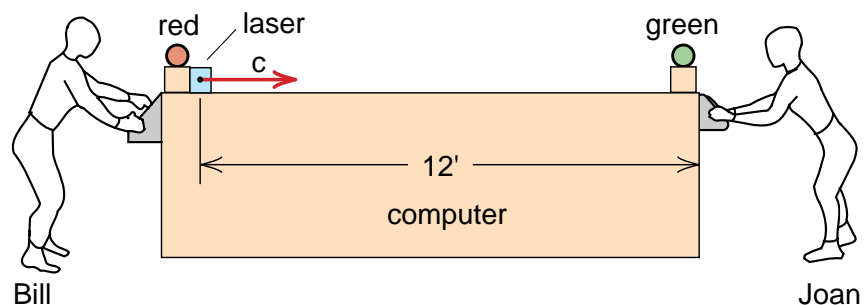
“You mean,” interrupts Joan, “that *you cannot violate causality if information does not travel faster than the speed of light?*”

“That’s right,” the instructor replies, “that’s one of the important and basic consequences of Einstein’s theory.”

“That’s interesting,” adds Bill. “It would violate the principle of relativity if we observed the astronaut’s capsule, or probably any other object, traveling faster than the speed of light. The speed of light is beginning to play an important role.”

Figure 31

If the answer to Bill's question takes 12 nanoseconds to travel through the 12 foot long computer, then this is the setup required to check the timing. The red bulb fires instantaneously, and everyone agrees that the red flash occurs first, and the answer appears later.



“That’s pretty far out,” replied Joan. “I didn’t know that physics could say anything about how information – *ideas* – moved.”

Jim, who had been sitting in the back of the classroom and not saying much, raised his hand. “At the beginning of the course when we were talking about sound pulses, you said that the more rigid the material, the faster the speed of sound in the material. You used Slinky pulses in your demonstrations because a Slinky is so compressible that a Slinky pulse travels slowly. You can’t compress air as easily as a Slinky, and sound pulses travel faster in air. Since steel is very rigid, sound goes very fast in steel.”

“During these discussions about the speed of light, I have been wondering. Is there any kind of material that is so rigid that sound waves travel at the speed of light?”

“What made you ask that?” the instructor asked.

“I’ve been reading a book about the life and death of stars,” Jim replied. I just finished the chapter on neutron stars, and they said that the nuclear matter in a neutron star was very incompressible. It had to be to resist the strong gravitational forces. I was wondering, how fast is the speed of sound in this nuclear matter?”

“Up close to the speed of light,” the instructor replied.

“If the nuclear matter were even more rigid, more incompressible, would the speed of sound exceed the speed of light?” Jim asked.

“It can’t,” the instructor replied.

“Then,” Jim asked, “doesn’t that put a limit on how incompressible, how rigid matter can be?”

“That looks like one of the consequences of Einstein’s theory,” the instructor replies.

“Then that explains what they were trying to say in the next chapter on black holes. They said that if you got too much matter concentrated in a small region, the gravitational force would become so great that it would crush the matter out of existence.”

“I didn’t believe it, because I thought that the matter would be squeezed down into a new form that is a lot more incompressible than nuclear matter, and the collapse of the star would stop. But now I am beginning to see that there may not be anything much more rigid than nuclear matter. Maybe black holes exist after all.”

“Will you tell us about neutron stars and black holes?” Joan asks eagerly.

“Later in the course,” the instructor responds.

APPENDIX A

Class Handout

To predict how long it takes for the trigger pulse to catch the red bulb in Figure (27b), let l be the uncontracted half length of the lecture bench (6 feet for our discussion). To the Martian, that half of the lecture bench has contracted to a length $l\sqrt{1 - v^2/c^2}$.

In the race, the red bulb traveling at a speed v , starts out a distance $l\sqrt{1 - v^2/c^2}$ ahead of the trigger pulse, which is traveling at a speed c . Let us assume that the race lasts a time t and that the trigger pulse catches the red bulb a distance x from where the trigger pulse started. Then we have

$$x = ct \quad (30)$$

In the same time t , the green bulb only travels a distance $x - l\sqrt{1 - v^2/c^2}$, but this must equal vt ;

$$vt = x - l\sqrt{1 - v^2/c^2} \quad (31)$$

Using Equation (30) in (31) gives

$$\begin{aligned} vt &= ct - l\sqrt{1 - v^2/c^2} \\ t(c - v) &= l\sqrt{1 - v^2/c^2} \end{aligned}$$

Solving for t gives

$$\begin{aligned} t &= \frac{l\sqrt{1 - v^2/c^2}}{c - v} = \frac{l\sqrt{(1 + v/c)(1 - v/c)}}{c(1 - v/c)} \\ &= \frac{l\sqrt{1 + v/c}}{c\sqrt{1 - v/c}} \end{aligned}$$

If v is very close to c , then $(1 + v/c) \approx 2$ and we get

$$t \approx \frac{l\sqrt{2}}{c} \left(\frac{1}{\sqrt{1 - v/c}} \right)$$

If we plug in the numbers $t = 3 \times 10^7$ seconds (one earth year), $l = 6$ feet, $c = 10^9$ feet/second, we get

$$\begin{aligned} \sqrt{1 - v/c} &= \frac{l\sqrt{2}}{ct} = \frac{6 \text{ ft } \sqrt{2}}{10^9 \text{ ft/sec} \times 3 \times 10^7 \text{ sec}} \\ &= 2.8 \times 10^{-16} \end{aligned}$$

Squaring this gives

$$(1 - v/c) = 8 \times 10^{-32}$$

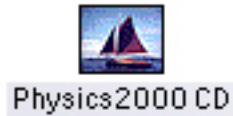
Thus if

$$\begin{aligned} v &= (1 - 8 \times 10^{-32})c \\ &= .9999999999999999999999999999992c \end{aligned}$$

then *the race will last a whole year*. On the other side, the trigger signal runs into the green flash bulb in far *less than a nanosecond* because the lecture bench is highly Lorentz contracted.

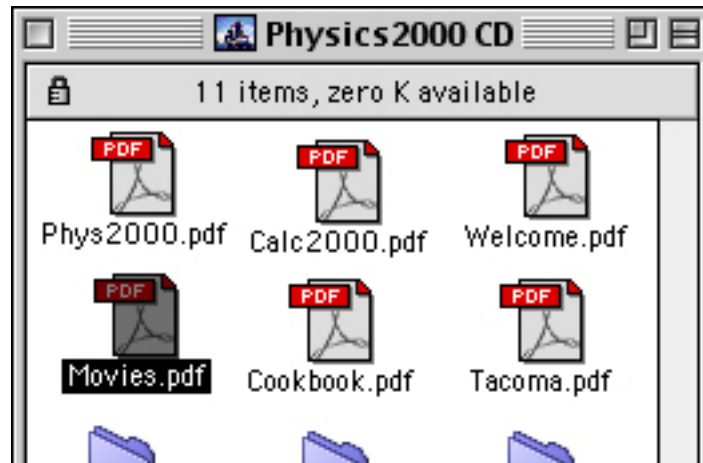
Finding the Muon Lifetime Movie on the *Physics2000* CD

1) Click on the Physics2000 CD icon



(Nineteenth Century Gaff Rigged Sloop)

2) Click on the Movies icon



3) Clicking on the picture starts the movie

Movies

Press the esc key to close out a movie

Click on the left side of a picture for a movie with 1x magnification. Click to the right for more magnification. Click on the descriptive text to take you to the page in the textbook where the movie is discussed.



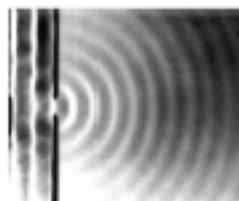
Muon Lifetime Movie
36 minute movie showing a real clock running slow.



UnDive
A dive with time running backward.



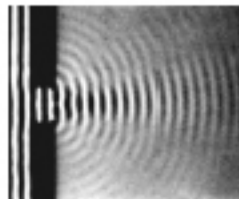
Conservation of Angular Momentum
She gets the wheel's angular momentum.



Huygens
Ripple tank Demonstration of the Huygens principle.



Bicycle Wheel Gyroscope
It doesn't fall like you would expect.



Wider Slit
Slit width is two wavelengths.