Chapter 33 non calculus
Quantum Mechanics III
The Uncertainty Principle – Time and Energy

In the last chapter we saw that if all particles had a particle-wave nature obeying the de Broglie relationship $p = h/\lambda$*, then there would be a fundamental restriction to the accuracy with which one could make experimental measurements. This limit is known as the uncertainty principle which comes in two forms represented by the equations*

$$\Delta p \Delta x \geq h$$

$$\Delta E \Delta t \geq h$$

The first form $\Delta p \Delta x \geq h$ *tells us that if we make a position measurement to within an accuracy* Δx*, we have made the momentum uncertain by an amount at least as large as* Δp*. The second form* $\Delta E \Delta t \geq h$ *tells us that if we have only a limited time* Δt *in which to make an energy measurement, the energy will be uncertain at least as much as* ΔE*.*

In the appendix to the last chapter, we derived the time energy form of the uncertainty principle from $\Delta p \Delta x \geq h$ *for a non relativistic particle where E was the kinetic energy* $1/2mv^2$*. We asked you to apply this to an electron dropped from rest to see that it would be impossible to accurately track the motion of an electron in the first 1/30 of a second.*

In this chapter we apply Fourier analysis to a short laser pulse and end up with a fully relativistic derivation of the uncertainty principle for the photons, or photon, in the pulse. In this derivation we more clearly see how the uncertainty principle is a direct consequence of the particle-wave nature of matter.

Perhaps the most interesting and important part of the chapter comes when we reconcile the time-energy form of the uncertainty principle with the law of conservation of energy. The result, which is totally outside the realm of classical physics, may have implications related to the origin of the universe.

A FEMTOSECOND LASER PULSE

In the last chapter, we used an analysis of the two slit electron diffraction experiment to introduce the position-momentum form of the uncertainty principle. In this chapter we will use an experiment involving the analysis of a pulsed laser beam to introduce the time-energy form.

Since the 1990s, it has been possible to build lasers that send out very short pulses, pulses only a few wavelengths long. Such pulses are useful for effectively taking "flash" pictures of the behavior of the electrons in chemical reactions.

The experiment we will discuss involves the pulse shown in Figure (1a). This is a graph of the intensity of the electric field in the pulse. The wave is called a ***femtosecond laser pulse*** because the main part of the pulse is about 20 femtoseconds (fs) long. (One femtosecond, fs, is 10^{-15} seconds, or one millionth of a nanosecond. In Figure (1a), the main part of the pulse starts at –10 fs and goes up to +10 fs.)

The second graph, Figure (1b), shows the spectrum of radiation in the pulse. The center frequency has a wavelength of 800 nanometers (nm) which is longer than the visible spectrum wavelengths that range from 300 nm for blue, up to 400 nm for red. Thus the pulse is in the longer wavelength ***infrared*** part of the spectrum.

A wave with an 800 nm wavelength λ has a period $T = \lambda/c$ of 2.67 femtoseconds. Thus in the range from –10 fs to + 10 fs, there should be about 7 cycles. In Figure (1a) you will count 14 maxima in this 20 fs range because you are looking at the intensity of the electric field. The intensity is proportional to the square of the field, and when you square a sine wave, you get two maxima per cycle.

What may be surprising about the spectrum in Figure (1b) is that the spectrum is not sharp. The range from 750 nm to 850 nm is a third as wide as the range from 400 nm to 700 nm for visible light. The lasers we used in Chapter 25 to study diffraction patterns had a distinct frequency. Send that laser light through a diffraction grating and you would get a sharp line rather than the spread out spectrum we see in Figure (1b).

One might be tempted to ask "What is wrong with the laser used in Figure (1a)?" "Why doesn't it give the pure single wavelength waves we expect from lasers?"

What is wrong is that the laser pulse is only a few wavelengths long. In the section after next, we will use Fourier analysis to demonstrate that such a short pulse must contain a spectrum of wavelengths. We will see that the spread in wavelengths is needed to cancel out the waves outside the pulse. In the meantime we will see that the spectrum in Figure (1b) is consistent with the time-energy form of the uncertainty principle.

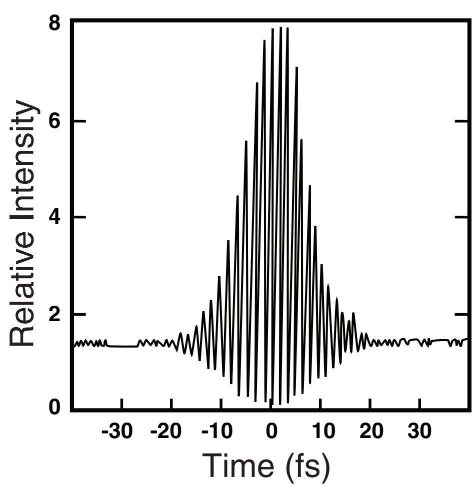


Figure 1a
Intensity of the electric field in a 20 femtosecond (fs) laser pulse. (From F. Hajiesmaeilbaigi and A. Azima Can. J. Phys. 76: *p498 (1998).)*

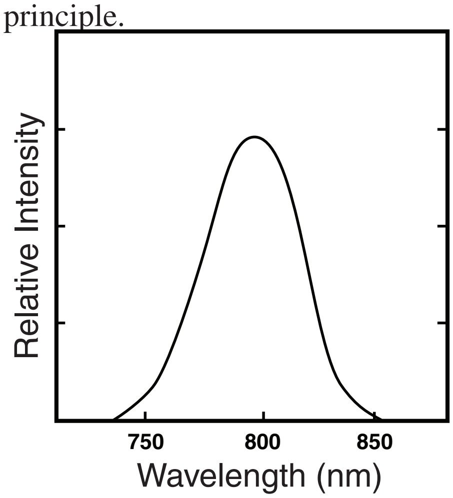


Figure 1b
Spectrum of the radiation in the laser pulse of Figure (1a). Visible light has a range from 400 nm to 700 nm, thus this spectrum, centered on 800 nm, is in the infrared.

USING THE UNCERTAINTY PRINCIPLE

As a laser pulse goes by us we have very little time, call it Δt, in which to measure the frequencies or energies of the photons in the pulse. The time-energy form of Heisenberg's uncertainty principle says that if only a short time Δt available for making an energy measurement, there will be an uncertainty ΔE in the result, where ΔE must be as large as $\Delta E = h/\Delta t$.

To apply the uncertainty principle to the laser pulse in Figure (1), we note that the pulse goes by in 20 fs. Thus we have only a time Δt = 20 fs in which to measure the spectrum of the laser light. But measuring the spectrum means measuring the frequencies or energies of the photons in the pulse. Thus measuring the spectrum is an energy measurement which must be uncertain by an amount $\Delta E = h/\Delta t$.

It is easier if we turn the argument around, first solving for the uncertainty ΔE in the energy of the photons in the pulse. Then we will use the uncertainty principle written as $\Delta t = h/\Delta E$ to see if we predict the correct length Δt of the pulse.

In Figure (1b) we see that the laser pulse consists of wavelengths ranging from as short as 750 nm up to 850 nm. The longer wavelength, lower frequency photons have a frequency f_- given by

$$f_- = \frac{c}{\lambda_-} = \frac{3\times 10^8 \text{m/sec}}{850\times 10^{-9}\text{m}} = .352\times 10^{15}\text{sec}^{-1} \tag{1a}$$

(The dimensions of frequency, cycles/sec, is is actually 1/sec or sec^{-1} because cycles are dimensionless.) The shorter wavelength photons have the higher frequency f_+ given by

$$f_+ = \frac{c}{\lambda_+} = \frac{3\times 10^8 \text{m/sec}}{750\times 10^{-9}\text{m}} = .400\times 10^{15}\text{sec}^{-1} \tag{1b}$$

From Einstein's photoelectric effect formula $E_{photon} = hf$ we see that photons in the pulse can have an energy as low as $E_- = hf_-$ up to an energy as high as $E_+ = hf_+$. We do not know what the energy of any given photon in the pulse is. All we know is that it is somewhere in the range between E_- and E_+.

We can say that the photon's energy is uncertain by an amount

$$\begin{aligned}\Delta E &= E_+ - E_- \qquad \textit{uncertainty of the energy of any photon in the pulse}\\ &= hf_+ - hf_-\\ &= h(.400 - .352)\times 10^{15}\text{sec}^{-1} \qquad (2)\\ &= h\times(.048)\times 10^{15}\text{sec}^{-1}\end{aligned}$$

Using the uncertainty principle in the form $\Delta t = h/\Delta E$ we get

$$\Delta t = \frac{h}{\Delta E} = \frac{h}{h\times(.048)\times 10^{15}\text{sec}^{-1}}$$

The factors of h cancel and we are left with

$$\boxed{\Delta t = 20.8\times 10^{-15}\text{sec} \approx 20 \text{ femtoseconds}} \tag{3}$$

which is the length of the pulse seen in Figure (1a). Thus the uncertainty principle, applied to the spectrum in Figure (1b) correctly predicts the length of the laser pulse.

The fact that Planck's constant cancelled in Equation (3) suggests that the uncertainty principle is something more than just some weird quantum effect. It suggests that it is a consequence of a more general behavior of waves. We will demonstrate that this is true by showing that the uncertainty principle is seen as a natural wave phenomena when we take the Fourier transform of a wave pulse.

PULSE FOURIER TRANSFORM

We have recently modified the MacScope II program so that we can easily study the Fourier transform of short wave pulses like the laser pulse in Figure (1a). We will use this section to explain how the ***pulse Fourier transform*** works. Then we will go on to see how the time-energy form of the uncertainty principle emerges as a consequence.

In Figure (2) we whistled into a microphone attached to MacScope, and recorded the wave shown. A whistle produces a fairly good sine wave. In that figure we have also selected one cycle of the sine wave. When we press the button labeled ***Fourier***, we get the standard Fourier analysis window shown in Figure (3).

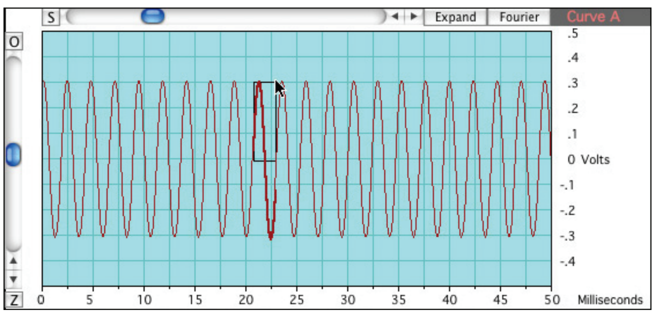


Figure 2
Selecting one cycle of a sine wave.

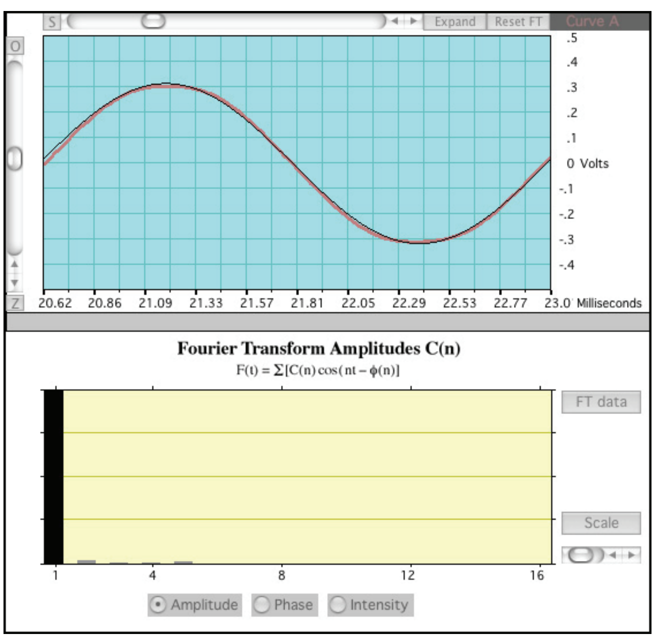


Figure 3
Fourier analysis of a sine wave. The assumption is that this cycle repeats indefinitely.

The way MacScope handles Fourier analysis is to assume that the selected section of the curve repeats indefinitely. In Figure (3) we selected one complete cycle of the sine wave. When that cycle is repeated indefinitely, we end up with a complete pure sine wave containing only the one frequency component which we see in the Fourier analysis window.

If, instead of pressing the ***Fourier*** button, we pressed the ***Pulse*** button of Figure (4a), we first get the window shown in Figure (4b) asking whether we want our pulse centered, left edge, or Gaussian. For this demonstration we chose ***Zero at Center*** and got the curve shown in Figure (5).

Now you can see what the ***Pulse Fourier Transform*** does. Instead of assuming that the selected section of the pulse repeats forever, the program zeros out all but the selected section of curve as shown in Figure (5). When we selected ***Zero at Center***, the program drew the zero line at the center of our selected section of the curve.

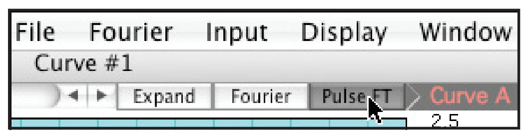


Figure 4a
Selecting the Pulse Fourier Transform.

Figure 4b
Choosing the Zero line.

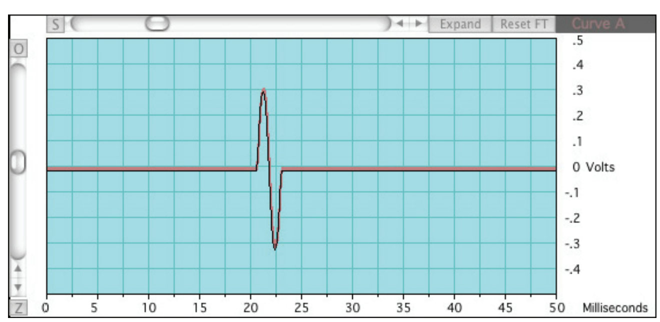


Figure 5
Selecting the Pulse Fourier Transform, which creates a pulse by zeroing all but the selected section of the curve.

Fourier Analysis of a Pulse

In Figure (6), we show the complete MacScope ***Curve A*** window. At the top we see our one cycle pulse with the rest of the curve zeroed out. Below in the Fourier analysis window we see a whole bunch of harmonics. Let us see where they came from.

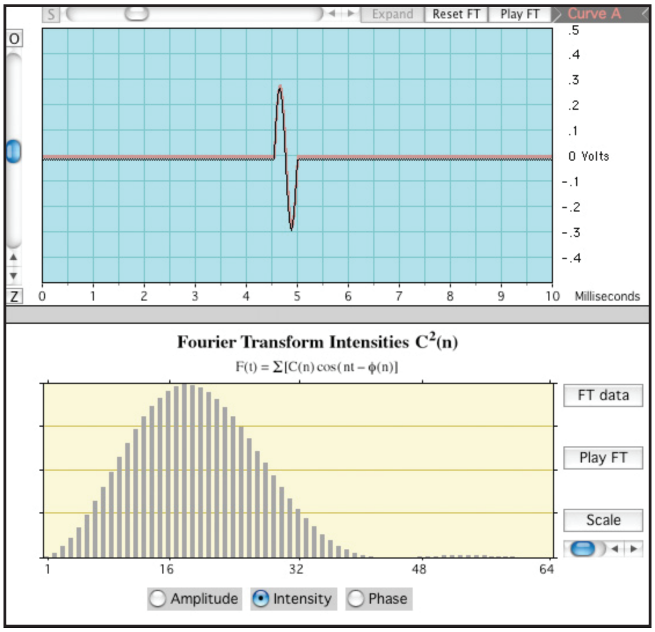
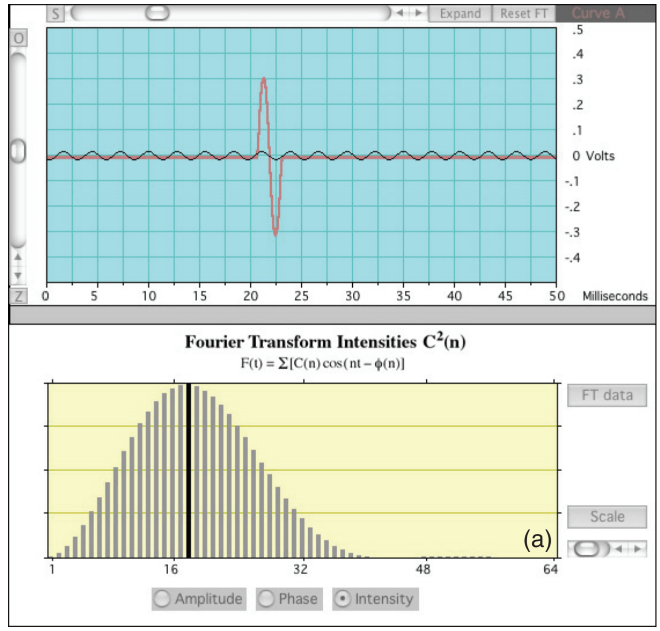


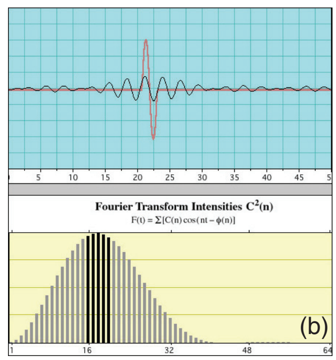
Figure 6
Harmonics in a short pulse.

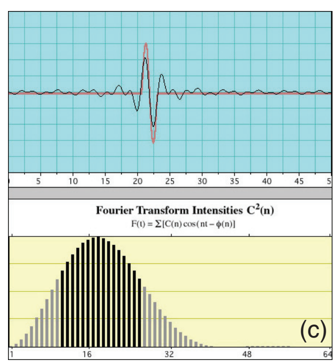
In Figure (7a) we have selected the center harmonic, and in the upper window see that MacScope has drawn a pathetic little sine wave that has the same wavelength as our selected cycle. This is telling us that we are not going to reconstruct our pulse from a single harmonic.

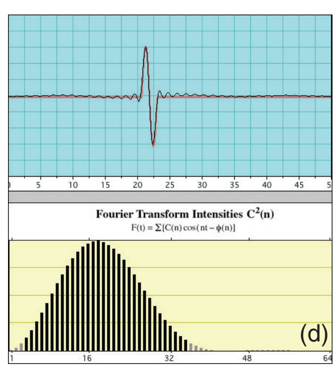
In Figure (7b) we selected the five biggest harmonics. In the upper window we see the result of adding these five sine waves together. They are beginning to build a pulse centered on our selected pulse.

In Figures (7c) and (7d) we see that as we select more harmonics, the closer we come to reconstructing our pulse. The lesson here is that to build a sharp pulse out of harmonic sine waves, takes a lot of sine waves. All these waves must add up at the pulse, and completely cancel each other elsewhere.









Figures 7
Fourier analysis of a pulse. Here we see how a short pulse is constructed from long sinusoidal waves.

In (a) we selected the largest harmonic and all it represents is a small sine wave.

When we add together the five biggest harmonics in (b), a pulse begins to form.

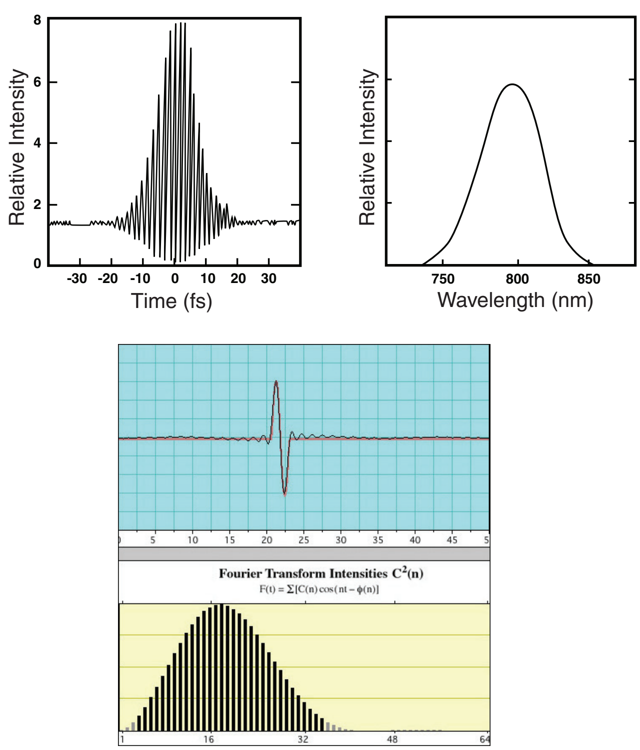
When we add up the 32 biggest harmonics, we get a close representation of the pulse in (d). We need a lot of harmonics to cancel the wave outside the pulse.

Intensity vs Amplitude

In our use of Fourier analysis so far, we have studied the ***amplitudes*** of the harmonics contained in a signal. But when we study the spectrum of light in a light wave, we measure the ***intensity*** of the electric field, which is the square of the field. If we want to compare the spectrum in the Fourier analysis of a short pulse, with the experimental spectrum of the laser pulse in Figure (1b), we should have MacScope plot the intensities rather than the amplitudes of the harmonics. This is easily done by looking at the buttons below the ***Fourier Analysis Plot***, and selecting the one labeled ***Intensity***, rather than the one labeled ***Amplitude***.

Spectrum of a Pulse

Now we are ready to compare Figure (7d) where we have selected most of the harmonics, with Figure (1b) showing the spectrum of wavelengths in our six cycle laser pulse. In both cases we see a similar spectrum of harmonics. You should now see that the reason why the short laser pulse contained a spectrum of wavelengths ranging from 750 nm up to 850 nm. It takes all these different wavelengths to add up to a short pulse.



Figures 1 & 7d
Comparison of the spectra of short pulses.

CHANGING Δt

At the beginning of the chapter, we used the uncertainty principle in the form $\Delta t = h/\Delta E$ to calculate the length Δt of the laser pulse. We found that with $\Delta E = h(f_+ - f_-)$ we got a time of Δt equal to 20 femtoseconds which accurately describes the length of the laser pulse.

What we want to do now is write the uncertainty principle equation in the form

$$\Delta E = \frac{h}{\Delta t} \tag{4}$$

and look at the consequences of changing the pulse length Δt.

The clear prediction of Equation (4) is that every time we double the length Δt of the pulse, we will cut the energy uncertainty ΔE in half. With MacScope, we can show that this is precisely what happens.

We can change the length of Δt by going back to our recording of a whistle, selecting more than one cycle of the sine wave, and doing a pulse Fourier transform. In Figure (8) we selected 2 cycles and see that we get a narrower spectrum.

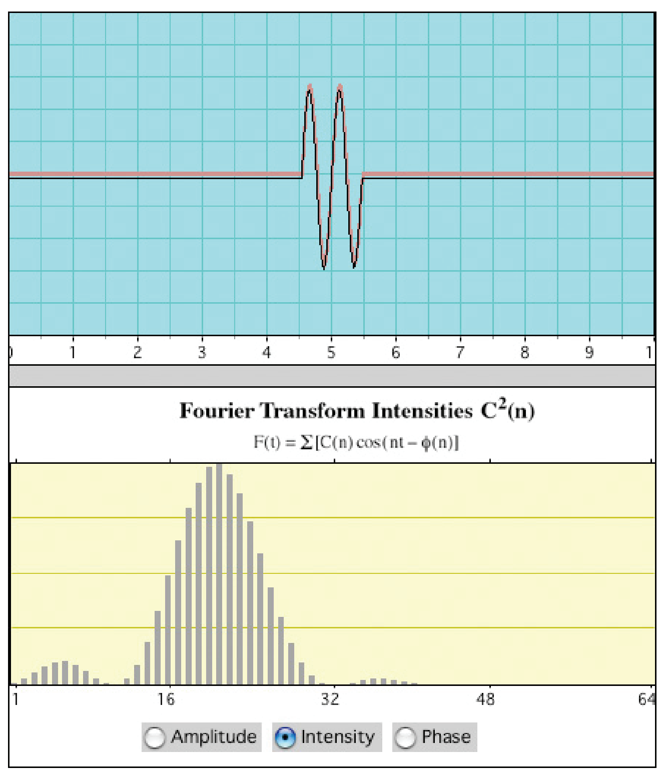


Figure 8
With 2 cycles the spectrum is narrower.

To get an overview of how the width of the spectrum of harmonics changes as we change Δt, in Figure (9) we kept doubling Δt from 1 cycle to 2 cycles to 4 cycles to 8 cycles. We see that the frequency spread in the main harmonics drops from 32 harmonics, to 16 harmonics, to 8 harmonics, to 4 harmonics. Clearly doubling the length of the pulse cuts the spread in harmonics in half.

The spread in harmonics tells us the range of frequencies $(f_+ - f_-)$ that are important in the pulse. Multiply this range of frequencies by Planck's constant h, and you get the formula for the range ΔE of photon energies in the pulse.

$$\Delta E = h(f_+ - f_-)$$

But this range of photon energies is the uncertainty in our knowledge of the energy of any one photon in the pulse. Thus we see that the uncertainty in the photon energy is cut in half every time we double the length of the pulse.

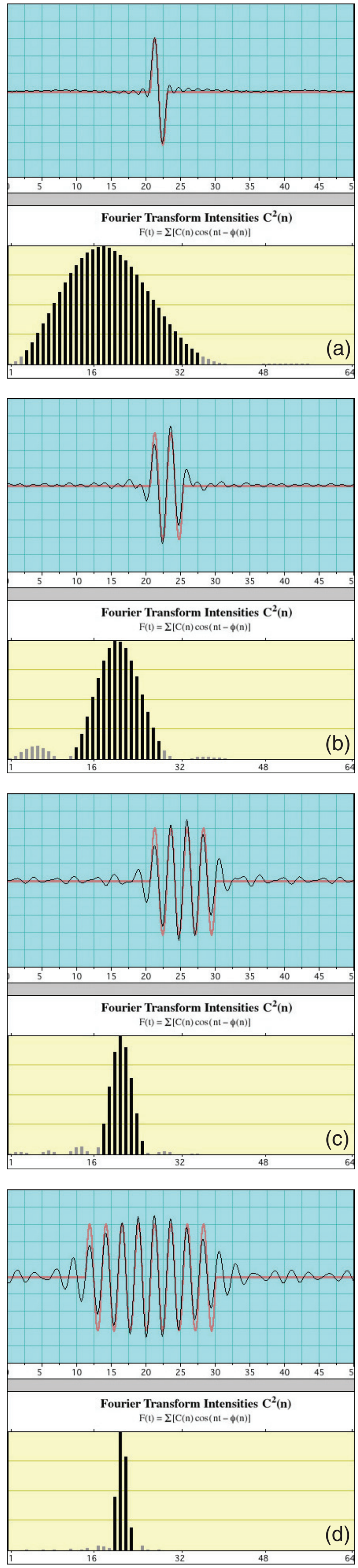
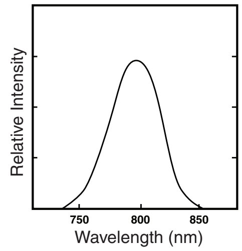


Figure 9
When we double the length Δt of the pulse, we cut the spread ΔE of the harmonics in half. The product $\Delta E \Delta t$ remains constant.

PROBABILITY INTERPRETATION

By now we should be quite comfortable with seeing the frequency or wavelength spectrum of the laser pulse in Figure (1b). The graph tells us that most of the photons have a wavelength around $\lambda = 800$ nm, while some photons have a wavelength as long as $\lambda_+ = 850$ nm, or as short as $\lambda_- = 750$ nm. If we change the horizontal scale from photon wavelengths to photon energies hc/λ, then we get a plot of the distribution of the energies of the photons in the pulse.



We can note that when we did a Fourier analysis of a pulse, we got a spectrum of the harmonic frequencies f_n. To get the spectrum of photon energies hf_n, you just multiply by Planck's constant h.

Now we are going to ask a question that, at first, may seem controversial. Suppose the laser pulse of Figure (1a) contained ***only one photon***. Then how should we interpret the spectrum in Figure (1b)? What does it mean to have a spectrum of energies for a single photon?

The answer to this question lies in the probability interpretation of the photon wave. If there were only one photon in the Figure (1) laser pulse, then ***the spectrum tells us the distribution of probabilities that the photon has the corresponding frequency or energy.*** What was an energy spectrum when we had many photons, becomes a probability spectrum when we have one or a few photons.

Even if there are many photons in the pulse, we can still interpret the spectrum as a probability spectrum for each photon. Then, for example, if each photon in the laser pulse is most likely to have a wavelength of around 800 nm, then when we measure photon wavelengths, we should find the most photons in the 800 nm wavelength range.

With the probability interpretation, any given photon has some probability of being at the high energy part of the spectrum, and some probability of being at the low energy end. Thus the width ΔE of the spectrum is truly the uncertainty in the photon's energy.

Measuring Short Times

We have said that pulsed lasers can produce pulses as short as 2 femtoseconds. How do we know that? Suppose we gave you the job of measuring the length of the laser pulse, and the best oscilloscope you had could measure times no shorter than a nanosecond. This is a million times too slow to see a femtosecond pulse. What do you do?

We answered that question at the beginning of this chapter. We used the uncertainty principle $\Delta t = h/\Delta E$ to correctly calculate the 20 femtosecond length of the Figure (1) pulse. Essentially we are using a diffraction grating as a clock!

Exercise 1

An electron is in an excited state of the hydrogen atom, either the second energy level at –3.40 eV, or the third energy level at –1.51 eV. You want to do an experiment to decide which of these two states the electron is in. What is the least amount of time you must take to make this measurement?

Short Lived Elementary Particles

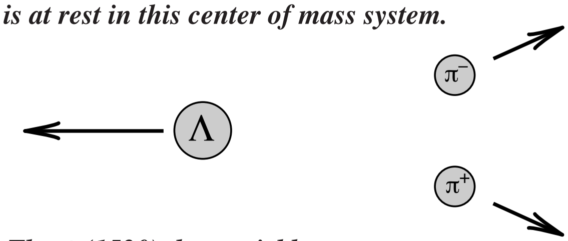
To drive home the fundamental nature of the relationship $\Delta E = h/\Delta t$, we will discuss an unstable elementary particle that lives for such a short time Δt that its rest mass energy m_0c^2 is demonstrably uncertain. We will then use the equation $\Delta t = h/\Delta E$ to measure the very short lifetime of the particle.

We usually think of the rest energy of a particle as having a definite value. For example the rest energy of a proton is $938.2723 \times 10^6 \text{eV}$. The proton itself is a composite particle made of 3 quarks, and the number 938.2723 MeV represents the total energy of the quarks in the allowed wave pattern that represents a proton. This rest energy has a very definite value because the proton is a stable particle with plenty of time to settle into a precise wave pattern.

A rather different particle is the so-called "Lambda (1520)" or "$\Lambda(1520)$", which is another combination of 3 quarks, but very short lived. The name comes partly from the fact that the particle's rest mass energy is about 1520 million electron volts (MeV). As indicated in Figure (10), a $\Lambda(1520)$ can be created as a result of the collision between a K^- meson and a proton.

a) A K^-meson and a proton are about to collide. We are looking at the collision in a coordinate system where the total momentum is zero (the so-called "center of mass" system).

b) In the collision a $\Lambda(1520)$ particle is created. It is at rest in this center of mass system.



c) The $\Lambda(1520)$ then quickly decays into a lower energy Λ particle and two π mesons.

Figure 10
A $\Lambda(1520)$ particle can be created if the total energy (in the center of mass system) of the incoming particles equals the rest mass energy of the $\Lambda(1520)$.

We are viewing the collision in a special coordinate system, where the total momentum of the incoming particles is zero. In this coordinate system, the resulting $\Lambda(1520)$ will be at rest. By conservation of energy, the total energy of the incoming particles should equal the rest mass energy of the $\Lambda(1520)$. Thus if we collide K^- particles with protons, we expect to create a $\Lambda(1520)$ particle only if the incoming particles have the right total energy.

Figure (11) shows the results of some collision experiments, where a K^- meson and a proton collided to produce a Λ and two π mesons. The probability of such a result peaked when the energy of the incoming particles was 1,520 MeV. This peak occurred because the incoming K^- meson and proton created a $\Lambda(1520)$ particle, which then decayed into an ordinary Λ and two π mesons, as shown in Figure (10). The $\Lambda(1520)$ was not observed directly, because its lifetime is too short.

Figure (11) shows that the energy of the incoming particles does not have to be exactly 1520 MeV in order to create a $\Lambda(1520)$. The peak is in the range from about 1510 to 1530 Mev, which implies that the rest mass energy of the $\Lambda(1520)$ is 1520 MeV plus or minus about 10 MeV. From one experiment to another, the rest mass energy can vary by about 20 MeV. (The experimentalists quoted a variation of 16 MeV.)

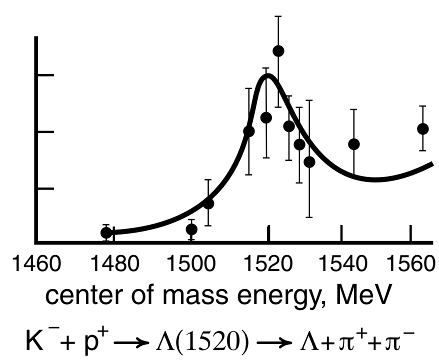


Figure 11

The probability that a K^-meson and a proton collide to produce a Λ particle and two π mesons, peak at an energy of 1520 MeV. The peak results from the fact that a $\Lambda(1520)$ particle was created and quickly decayed into the Λ and two π mesons. The probability peaks at 1520 MeV, but can be seen to spread out over a range of about 16MeV. The small circles are experimental values, the vertical lines represent the possible error in the value. (Data from M.B. Watson et al., *Phys. Rev.* 131(1963).)

Why isn't the peak sharp? Why does the rest mass energy of the $\Lambda(1520)$ particle vary by as much as 16 to 20 MeV from one experiment to another? The answer lies in the fact that ***the lifetime of the Λ(1520) is so short, that the particle does not have enough time to establish a definite rest mass energy***. The 16 MeV variation is the uncertainty ΔE in the particle's rest mass energy that results from the fact that the particle's lifetime is limited.

The uncertainty principle relates the uncertainty in energy ΔE to the time Δt available to establish that energy. To establish the rest mass energy, the time Δt available is the particle's ***lifetime***. Thus we can use the uncertainty principle to estimate the lifetime of the $\Lambda(1520)$ particle. With $\Delta t = h/\Delta E$ we get

$$\Delta t = \frac{h}{\Delta E} = \frac{6.63 \times 10^{-34}\text{joule second}}{16{\times}10^{6}\text{eV} \times 1.6{\times}10^{-19}\dfrac{\text{joule}}{\text{eV}}}$$

$$\Delta t = 2.6 \times 10^{-22}\text{ seconds} \tag{5}$$

The lifetime of the $\Lambda(1520)$ particle is of the order of 10^{-22} seconds! This is only about 10 times longer than it takes light to cross a proton! Only by using the uncertainty principle could we possibly measure such short times.

THE UNCERTAINTY PRINCIPLE AND ENERGY CONSERVATION

The fact that for short times the energy of a particle is uncertain, raises an interesting question about basic physical laws like the law of conservation of energy. If a particle's energy is uncertain, how do we know that energy is conserved in some process involving that particle? The answer is — ***we don't***.

One way to explain the situation is to say that nature will cheat if it can get away with it. Energy does not have to be conserved if we cannot do an experiment to demonstrate a lack of conservation of energy.

Consider the process shown in Figure (12). It shows a red, 2 eV photon traveling along in space. Suddenly the photon creates a positron-electron pair. The rest mass energy of both the positron and the electron are .51 MeV. Thus we have a 2 eV photon creating a pair of particles whose total energy is 1.02×10^{6}eV, a huge violation of the law of conservation of energy. A short time later the electron and positron come back together, annihilate, leaving behind a 2 eV photon. This is an equally huge violation of the conservation of energy.

But have we really violated the conservation of energy? During its lifetime, the positron-electron pair is a composite object whose total energy is uncertain. If the pair lived a long time, its total energy would be close to the expected energy of 1.02×10^{6}eV. But suppose the pair were in existence only for a very short time Δt, a time so short that the uncertainty in the energy could be as large as 1.02×10^{6}eV. Then there is some probability that the energy of the pair might be only 2 eV and the process shown in Figure (12) could happen.

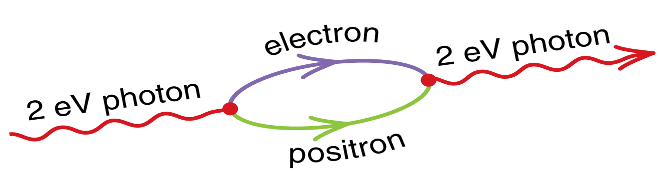


Figure 12
Consider a process where a 2 eV photon suddenly creates a positron-electron pair. A short time later the pair annihilates, leaving a 2 eV photon. In the long term, energy is conserved.

The length of time Δt that the pair could exist and have an energy uncertain by 1.02 MeV is

$$\Delta t = \frac{h}{\Delta E} = \frac{6.63 \times 10^{-34}\text{joule sec}}{1.02\times 10^{6}\text{eV}\times 1.6\times 10^{-19}\dfrac{\text{joule}}{\text{eV}}}$$

$$\Delta t = 4 \times 10^{-21}\text{sec} \qquad (6)$$

Another way to view the situation is as follows. Suppose the pair in Figure (12) lasted only 4×10^{-21} seconds or less. Even if the pair had an energy of $1.02 \times 10^{6}\text{eV}$, the lifetime is so short that any measurement of the energy of the pair would be uncertain by at least $1.02 \times 10^{6}\text{eV}$, and the experiment could not detect the violation of the law of conservation of energy. In this point of view, if we cannot perform an experiment to detect a violation of the conservation law, then the process should have some probability of occurring.

Does a process like that shown in Figure (12) actually occur? If so, is there any way that we can know that it does? The answer is yes, to both questions. It is possible to make extremely accurate studies of the energy levels of the electron in hydrogen, and to make equally accurate predictions of the energy using the theory of ***quantum electrodynamics***.

We can view the binding of the electron to the proton in hydrogen as resulting from the continual exchange of photons between the electron and proton. During this continual exchange, there is some probability that the photon creates a positron-electron pair that quickly annihilates as shown in Figure (12). In order to predict the correct values of the hydrogen energy levels, the process shown in Figure (12) has to be included. Thus we have direct experimental evidence that for a short time the particle-antiparticle pair existed.

QUANTUM FLUCTUATIONS AND EMPTY SPACE

We began the text with a discussion of the principle of relativity—that you could not detect your own motion relative to empty space. The concept of empty space seemed rather obvious—space with nothing in it. But the idea of empty space is not so obvious after all.

With the discovery of the cosmic background radiation, we find that all the space in this universe is filled with a sea of photons left over from the big bang. We can accurately measure our motion relative to this sea of photons. The earth is moving relative to this sea at a velocity of 600 kilometers per second toward the Virgo cluster of galaxies. While this measurement does not violate the principle of relativity, it is in some sense a measurement of our motion relative to the universe as a whole.

Empty space itself may not be empty. Consider a process like that shown in Figure (13) where a photon, an electron, and a positron are all created at some point in space. A short while later the three particles come back together with the positron and electron annihilating and the photon being absorbed.

One's first reaction might be that such a process is ridiculous. How could these three particles just appear and then disappear? To do this we would have to violate both the laws of conservation of energy and momentum.

But, of course, the uncertainty principle allows us to do that. We can, in fact, use the uncertainty principle to estimate how long such an object could last. The arguments would be similar to the ones we used in the analysis of the process shown in Figure (12).

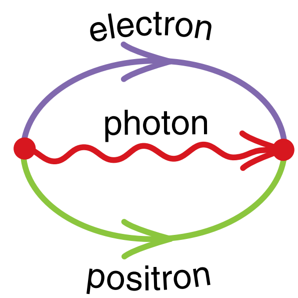


Figure 13
Quantum fluctuation. The uncertainty principle allows such an object to suddenly appear, and then disappear.

In the theory of quantum electrodynamics, a completely isolated process like that shown in Figure (13) does not affect the energy levels of the hydrogen atom and should be undetectable in electrical measurements. But such a process might affect gravity. A gravitational wave or a graviton might interact with the energy of such an object. Some calculations have suggested that such interactions could show up in Einstein's theory of gravity (as a contribution to his famous ***cosmological constant).***

An object like that shown in Figure (13) is an example of what one calls a ***quantum fluctuation***. Here we have something that appears and disappears in so-called empty space. If such objects can keep appearing and disappearing, then we have to revise our understanding of what we mean by empty.

The uncertainty principle allows us to tell the difference between a quantum fluctuation and a real particle. A quantum fluctuation like that in Figure (13) violates conservation of energy, and therefore cannot last very long. A real particle can last a long time because energy conservation is not violated.

However, there is not necessarily that much difference between a real object and a quantum fluctuation. To see why, let us take a closer look at the π meson. The π^+ is a particle with a rest mass energy of 140 MeV, that consists of a quark-antiquark pair. The quark in that pair is the so-called ***up*** quark that has a rest mass of roughly 400 MeV. The other is the ***antidown*** quark that has a rest mass of about 700 MeV. (Since we can't get at isolated quarks, the quark rest masses are estimates, but should not be too far off). Thus the two quarks making up the π meson have a total rest mass of about 1100 MeV. How could they combine to produce a particle whose rest mass is only 140 MeV?

The answer lies in the potential energy of the ***gluon force*** that holds the quarks together. As we have seen many times, the potential energy of an attractive force is negative. In this case the potential energy of the gluon force is almost as big in magnitude as the rest mass of the quarks, reducing the total energy from 1100 MeV to 140 MeV.

Suppose we had an object whose negative potential energy was as large as the positive rest mass energy. Imagine, for example, that the object consisted of a collection of point sized elementary particles so close together that their negative gravitational potential energy was the same magnitude as the positive rest mass and kinetic energy. Suppose such a collection of particles were created in a quantum fluctuation. How long could the fluctuation last?

Since such an object has no total energy, the violation ΔE of energy conservation is zero, and therefore the lifetime $\Delta t = h/\Delta E$ could be forever.

Suppose the laws of physics required that such a fluctuation rapidly expand, greatly increasing both the positive rest mass and kinetic energy, while maintaining the corresponding amount of negative gravitational potential energy. As long as ΔE remained zero, the expanding fluctuation could keep on going. Perhaps such a fluctuation occurred 13.7 billion years ago and we live in it now.

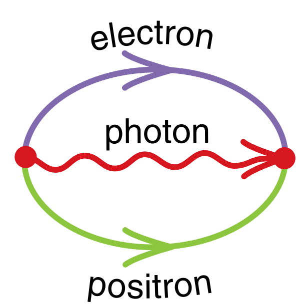


Figure 13 (repeated)
Quantum fluctuation. The uncertainty principle allows such an object to suddenly appear, and then disappear.

CHAPTER 33 REVIEW

This is one of the most important chapters in the text, because it gives an entirely new perspective on the basic nature of physics. In a sense, many of the previous chapters have been included in order to provide the background needed for this chapter.

We began the chapter with the experiment of Figure (1) which showed that a short laser pulse, about 12 cycles long, contained a broad spectrum of wavelengths. The pulse was in the infrared, and the spread of wavelengths, from 750 to 850 nanometers was a third as wide as the spread of visible wavelengths which range from 400 to 700 nanometers.

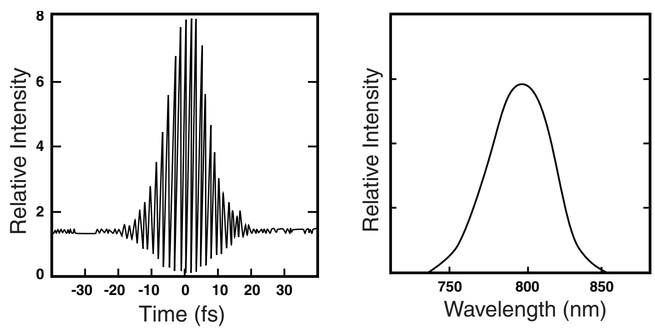


Figure 1

One usually thinks of a laser as having a pure color, like the red lasers that are so common. But the spread in wavelengths of the laser pulse was due not to poor equipment but to a fundamental property of waves. When we used MacScope to study the Fourier transform of a short pulse, we saw that any short pulse had to be made up from a spread of wavelengths or frequencies. The reason for this is that a sine wave is an infinitely long wave. If you are going to construct a short pulse out of sine waves, you need a distribution of wavelengths to get complete cancellation beyond the ends of the pulse.

Figure (9) shows the main feature of the spread in frequencies required to create a short pulse. As we go down from (9a) to (9d), we keep doubling the length of the pulse and see that the spread in frequencies is cut in half. In other words, if we call Δt *the length of the pulse, and* Δf *the width of the spread in frequencies, the product* $\Delta t \Delta f$ *does not change as we keep doubling* Δt.

Let us look at the product $\Delta t \Delta f$ *for the laser pulse in Figure (1). Since the pulse is 20 femtoseconds long, we have for the length of the pulse*

$$\Delta t = 20 \times 10^{-15} \text{ seconds}$$

The wavelengths range from $\lambda_- = 750 \times 10^{-9}$ *meters to* $\lambda_+ = 850 \times 10^{-9}$ *meters. The related frequencies are*

$$f_+ = \frac{c}{\lambda_-} = \frac{3 \times 10^8}{750 \times 10^{-9}} = 4.0 \times 10^{14} \text{sec}^{-1}$$

$$f_- = \frac{c}{\lambda_+} = \frac{3 \times 10^8}{850 \times 10^{-9}} = 3.5 \times 10^{14} \text{sec}^{-1}$$

and the spread in frequencies is

$$\Delta f = f_+ - f_- = 0.5 \times 10^{14} \text{sec}^{-1}$$

Thus the product $\Delta t \Delta f$ *has the surprisingly simple result*

$$\Delta t \Delta f = 20 \times 10^{-15} \text{sec} \times 0.5 \times 10^{14} \frac{1}{\text{sec}} = 1.0$$

As a final step, note that the laser pulse is made up of photons that obey Einstein's formula E = hf. Thus hΔf *is the energy spread* ΔE *of the photons in the laser pulse. With* $\Delta E = h\Delta f$ or $\Delta f = \Delta E/h$, we get

$$\Delta f \Delta t = \frac{\Delta E}{h}\Delta t = 1 \qquad \boxed{\Delta E \Delta t = h}$$

for the photons in the pulse.

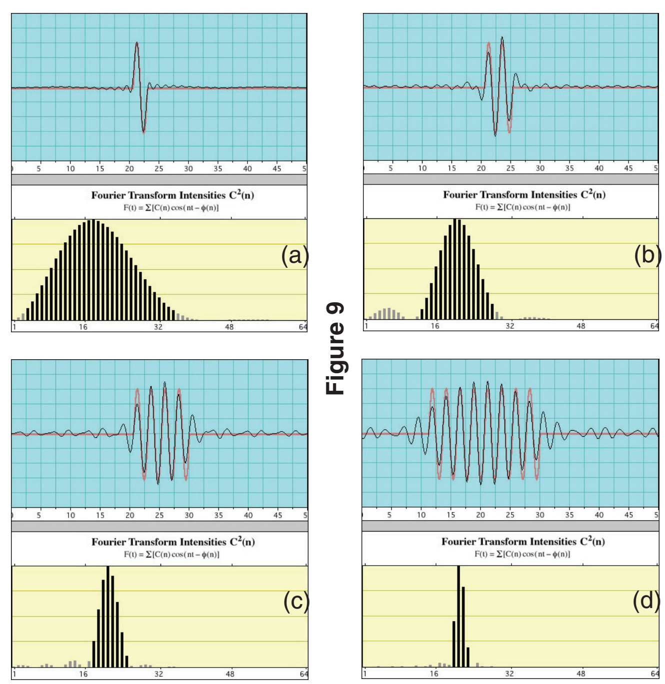
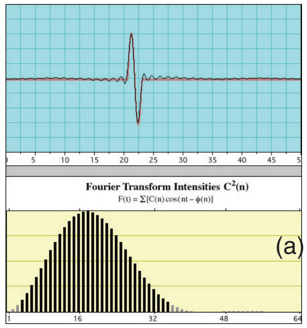


Figure 9

Probability Interpretation



Let us take another look at the short pulse in Figure (9a), which is made up of a fairly wide distribution of frequencies. If this represented a real laser pulse with many photons, the bottom graph would represent the **distribution of frequencies or energies** *of the photons in the pulse.*

Now comes the interesting question. Suppose there is **only one photon** *in the pulse. What is the meaning of the distribution of frequencies or energies mean for a single photon? The answer is that it is the* **distribution of probabilities** *that the photon has the corresponding frequency or energy. There is some probability that the photon has a frequency or energy as low as the first few harmonics shown. And some probability that its energy is as high as that represented by the 32nd harmonic. The photon's energy can be anywhere in this range of frequency or energy. Thus our calculation of ΔE represents the uncertainty in the energy of the photon.*

If you read Figure (9) from (9d) to (9a), you see that as you make the laser pulse shorter, the uncertainty ΔE of the energy of the photon in the pulse becomes greater. The shorter the time Δt we have available to observe the photon, the more uncertain our measurement of the photon's energy becomes.

Energy Conservation

We introduced the concept of energy by saying that its main property was that it was conserved. Energy conservation became one of the most important laws in the text. Now we see that if you try to measure the energy of an object, be it the non relativistic particle discussed at the end of the last chapter, or a fully relativistic particle like a photon in a laser pulse, any measurement of the particle's energy will lead to uncertain results. How do we know that energy is conserved if we cannot measure it precisely?

Nature's answer to this question appears to be— ***energy only has to be conserved when you can measure it****.*

The most convincing evidence for this point of view comes from a detailed calculation of the energy of the electron in the hydrogen atom. In the theory of Quantum Electrodynamics, the electric force holding the electron in the atom is caused by the exchange of photons between the electron and the proton. Since the electron binding energy is only 13.6 eV, the photons on the average must have a small energy, not much larger than 13 eV.

But, the calculation shows that there is some finite probability for one of the photons to create an electron-positron pair during an exchange. Since the rest mass energy of a positron-electron pair is one million electron volts, we have a situation where a photon whose energy should be only a few electron volts creates a million electron volt pair.

How can this happen? The answer is that the pair lasts for such a short time Δt that the violation of energy conservation ΔE cannot be directly detected due to the uncertainty principle.

The uncertainty principle can also be used as a clock to measure very short times. We found that the rest mass of the so called (Λ1520) *particle varied by as much as 16 million electron volts when the particle decayed. The reason for this variation is that the particle did not live long enough to accurately determine its own rest mass energy. Calling this variation the uncertainty ΔE of the particle's rest mass energy, and Δt its lifetime, we got*

$$\Delta t = \frac{h}{\Delta E} = \frac{6.63\times 10^{34}\text{ joule sec}}{10^{6}\text{eV} \times 1.6\times 10^{-19}\text{ joule/eV}}$$
$$= 4\times 10^{-21}\text{ seconds}$$

There is no other way to measure such short times.

Quantum Fluctuations

The uncertainty principle affects our view of what we used to call empty space. To say that I have a region of empty space, is to say that that region has zero energy exactly. But if I do an experiment to measure the energy in that region of space, my answer must be uncertain by an amount $\Delta E = h/\Delta t$. Thus for short times, something is probably there.

A candidate is the example of a quantum fluctuation shown in Figure (13). To have an electron, a positron, and a photon, suddenly appear in space, requires a violation ΔE of the conservation of energy of at least one million eV. But if these particles all annihilate each other in a time $\Delta t = h/\Delta E$, the violation of conservation of energy cannot be observed and there should be some probability of such a process occuring.

The Early Universe

Recent studies of the early universe suggest that quantum flucuations are responsible for the diversity of structure we now see in the universe.

In Chapter 26 on photons, we discussed the three degree radiation that was released when the universe became transparent at the young age of 1/3 of a million years. (The universe is now 13.7 billion years old.) The main feature of this radiation is that it is very uniform. It took specially designed satellites to detect any variation from point to point in the sky.

The latest satellite (NASA's Wilkinson Microwave Anisotropy Probe [WMAP]) detected a pattern of small variations that appear to have resulted from quantum fluctuations in the very, very early universe. The current theory is that once the universe became transparent, these small variations allowed the gravitational collapse of clouds of gas to form the stars, galaxies, and the clusters of galaxies we see today. If the universe had been perfectly uniform, with no lumps created by quantum fluctuations, the universe would not have the structure we observe.

CHAPTER EXERCISE

Exercise 1 0n page 8

An electron is in an excited state of the hydrogen atom, either the second energy level at –3.40 eV, or the third energy level at –1.51 eV. You want to do an experiment to decide which of these two states the electron is in. What is the least amount of time you must take to make this measurement?

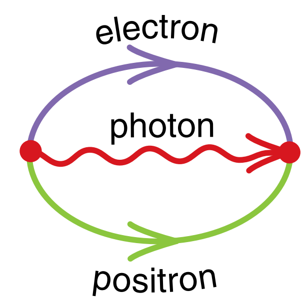


Figure 13 (repeated again)
Quantum fluctuation. The uncertainty principle allows such an object to suddenly appear, and then disappear.